TREATISE ON GRAVITY AND GRAVITOMAGNETISM
TREATE DE GRAVITE ET DE GRAVITOMAGNETISME

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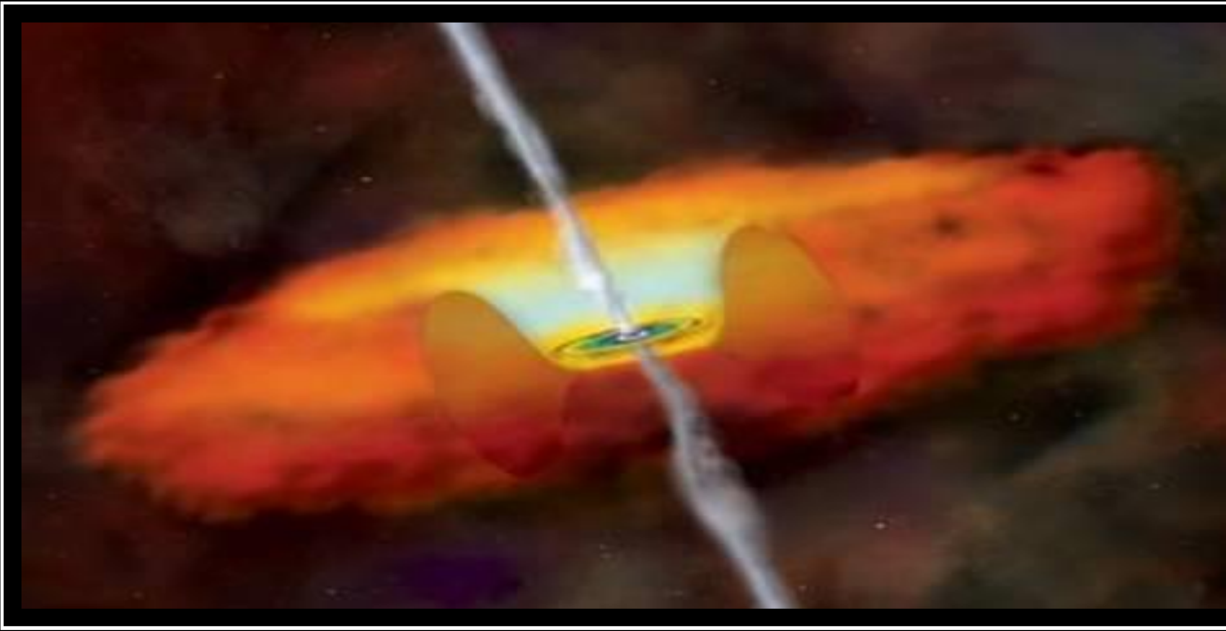


ABSTRACT

By honest observation, analogy and intuition the laws that govern physical phenomena can be deduced, this approach is know as the frontier physics evidence. The obtained laws should be in conformity with all the scientific observations that are accepted by the international scientific community. A central concept in science and the scientific method is that all evidence must be empirical, or empirically based, that is, dependent on evidence that is observable by the senses. Such methods are opposed to theoretical ab initio methods which are purely deductive and based on first principles that could lead to incoherent theories, maths is just a tool that helps us to have a rational approach to frontier physics but the approach to frontier physics must be guided by evidence that is observable by the senses, in our case, it is the astronomical observations that should guide us in validating the Treatise on Gravity and Gravitomagnetism.

We shall not attempt do define the nature of gravity but we shall define the laws that govern the remote forces of gravity by elaborating the Treatise on Gravity and Gravitomagnetism at far area interaction. In the coming years we shall attempt to define the near area electric and gravitational field interaction in order to explain electron forbidden bond, spectrometer characteristics and the co-habitation of protons in the atomic nuclease. The near area interaction does not obey the inverse square law; it probably obeys a wave like curve enveloped by pseudo inverse square law.

We know that the electrostatic interaction inverse square law (Law of Coulomb) and the gravitational interaction inverse square law (Law of Newton) are both governed by the same geometrical law in a static regime; we know that the electrical and the gravitational interaction are propagated at a finite speed. In pure mathematics, the inverse square law corresponds to the “dilution” of a field by the increase of the surface area of a sphere $4\pi r^2$, sound and electromagnetic waves power obeys this “dilution” law. The surface area of a sphere is the derivative of the volume of a sphere $4\pi r^3/3$ with respect to the radius r. We know that in computer science an CMOS AND gate is composed of transistors in series, this is equivalent to two water taps in series and an CMOS OR gate is composed of transistors in parallel, this is equivalent to two water taps in parallel, the truth table of the electric current gates is the same as the truth table of the masse (water) current water tap gates. In the first case we have an electric current and the second case we have a masse (water) current. Electronic and mechanical oscillators are both governed by the same laws (simple harmonic motion). Where the inductance is equivalent to inertia and capacity is equivalent to the spring elasticity (the elasticity of a diaphragm fixed in the cross section of a ring tube filled with a liquid). The electrical resistance is

equivalent a constriction in a liquid tube (medical thermometer use this constriction principle to enable accurate readings). Ignoring losses, the sum of inertia and elastic (inductance and capacity) energy is constant in simple harmonic motion system. By ignoring the symbolic physical quantities and constants, we know that there is no difference between an electrical system and a mechanical system, especially in simple harmonic motion. The two systems are equivalent; Fourier Transformation (Laplace Transformation as in MATLAB software) is applicable on both systems. Since Lorentz Transformation is applicable to the electric field, it is logical to think that Lorentz Transformation should be applicable on the gravitational field. In order to cross the threshold of the frontier physics evidence, our honest approach should be validated by astronomical observations because this is the only true basis in modern scientific methods.

Our approach is in accordance with the model published by Maxwell in 1891, in his third edition of Treatise on Electricity and Magnetism. It is by honest analogy to mechanics that Maxwell elaborated the Treatise on Electricity and Magnetism. By analogy to Treatise on Electricity and Magnetism done by Maxwell, it is logical to elaborate the Treatise on Gravity and Gravitomagnetism by using Lorentz Transformation as a mathematical tool without forgetting the specificity of gravity, thus masses of the same sign attract each other. By symmetry, logically masses of opposite signs should repulse each other but this is just a conjecture.

We know that in a dynamic regime, the so-called magnetic field is just a relativistic effect due to distortion of the electric field orthogonal to the relative movement of a charged particle with respect to an observer. It would be logical to think that a remote force in a dynamic regime, which we shall call gravitomagnetic field is just a relativistic effect due to distortion of the gravitational field orthogonal to the relative movement of a masse particle with respect to an observer. This remote force, thus gravitomagnetic field does not have the same nature as the magnetic field but should logically be governed by the same geometrical laws as the magnetic field in a dynamic regime. This honest approach is a major step stone towards the frontier physics evidence.

We know that science is not a dogma; if we agree on that point then we have the right to question the validity of the Big Bang theory. By using relativistic Doppler effect we show that the gamma rays bursts and red shift cosmic microwave background radiation are part of the frontier physics evidence. This does not necessary mean that the Universe is expanding. At $\upsilon = 95\%$ the speed of light, the probability of detecting red shift is more that 70% and for very low speeds above zero ($\upsilon > 0$), the probability is more than 50%. The red shift is necessary but not sufficient to support the Big Bang theory.

We know that the Einstein equation of masse energy equivalence is given by; $E = mc^2$, we deduce the momentum of an photon is equal to mc = E/c (masse times velocity) and with the help of the kinetic or collision theory of gasses elaborated by Maxwell as a mathematical tool, we elaborate the light dynamics thereby obtaining an accurate light radiation pressure equation that includes the coefficient of reflection. We then postulate the possibility of devising a light rocket engine that could propel us to the stars.

In accordance with the Treatise on Gravity and Gravitomagnetism, a gravity transversal wave generator of type stretch C has been devised and tested in Toulouse (29 September 2007). An independent laboratory will validate this gravity transversal wave generator.

Since the quasi stationary orthodox gravity theories do not offer a global and coherent explanation concerning gravity perturbations, can there be a physical science work of more importance than obtaining an understanding of these perturbations and seeking interaction with the remote forces of gravity? The facts are there, the facts remain the keystone in which the stability of a theory must be tested.

Much has been said about gravitodynamics, gravitomagnetism, black holes thermodynamics, and Big Bang theory. The Treatise on Gravity and Gravitomagnetism gives a global Cartesian and coherent quantitative analysis approach that is in conformity with the astronomical observations and that could be fed in a simulator ready to use.

When we shall have time, we shall write free software in C++. We can create distributed objects in a teamwork via Internet controlled by clock tics to simulate cosmological invents.

The Treatise on Gravity and Gravitomagnetism has determined the gravitodynamics **New Newton's Law** by taking into account the relativistic velocity of the interacting masse particles contrary to the quasi-stationary gravity law of Newton. The Treatise on Gravity and Gravitomagnetism and Doppler effect explain the following 9 cosmological blunders of the last 85 years in a Cartesian way;

1) Big Bang theory incoherencies by relativistic Doppler effect, Treatise on Gravity and Gravitomagnetism, gravitodynamics.
2) Mercury perihelion advance by Treatise on Gravity and Gravitomagnetism, gravitodynamics.
3) The Allais Effect - gravity waves by Treatise on Gravity and Gravitomagnetism, gravitodynamics.
4) The pioneer anomaly by Treatise on Gravity and Gravitomagnetism, gravitodynamics.
5) The galaxy disk shape flatness by Treatise on Gravity and Gravitomagnetism, gravitodynamics.
6) The spiral form aspiration of matter by the accretion disk by Treatise on Gravity and Gravitomagnetism, gravitodynamics.
7) The matter bipolar jets trajectory by Treatise on Gravity and Gravitomagnetism, gravitodynamics.
8) Galaxy rotation curve flatness by Treatise on Gravity and Gravitomagnetism, gravitodynamics.
9) The source of matter bipolar jets by Treatise on Gravity and Gravitomagnetism, gravitodynamics.

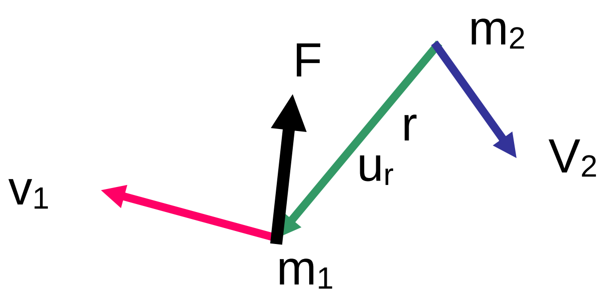
European Space Agency Thanks to the team for its experimental works that confirmed gravitomagnetism; Dr Martin Tajmar and Dr Clovis J. de Matos. I would like to thank Thierry de Mees and D.Turanyamin for their works in gravitomagnetism. Thanks to my compatriot (KENYA), **2004 Nobel Peace Prize Winner Wangari Maathai** for her dedication in the promotion of women rights and environment conservation.

Summary of Treatise on Gravity and Gravitomagnetism

Linear vector gravity

New Newton's Law

Gravitomagnetism masse law



The force exerted on a particle of masse m_1 moving at a velocity of V_1 by a particle of masse m_2 moving at a velocity of V_2 and both separated by a distance r with a unit vector u_r along r is given by;

$$F = -\frac{Gm_1m_2}{r^2}u_r + \frac{\mu_g m_1 m_2 V_1\times(V_2\times u_r)}{r^2} \quad (kg.m.s^{-2})$$

Where G ($m^3/kg.s^2$) is the constant of gravity and μ_g (m/kg) is the vacuum permeability of gravitomagnetic field. **We notice that the force F is not radial as described by Newton's law of gravity.**
$V_1\times(V_2\times u_r)$ is a vector product, the $(V_2\times u_r)$ vector product must be done first because the triple vector product is not associative.

If the particles are charged the force exerted on a particle of masse m_1 and of a charge of q_1 moving at a velocity of V_1 by a particle of masse m_2 and of a charge of q_2 moving at a velocity of V_2 is given by;

Unified gravitomagnetism and electromagnetism law

$$F = \left(-\frac{Gm_1m_2 + kq_1q_2}{r^2}\right)u_r + \left(\frac{\mu_g m_1m_2 + \mu_0 q_1q_2}{r^2}\right)V_1\times(V_2\times u_r)$$

Where $k = 1/4\pi\varepsilon_0$, ε_0 ($Kg/A^2.s^2$)is the vacuum permittivity of the electric field and μ_0 ($A^2.s^2/kg.m$) is the vacuum permeability of magnetic field.

Gravitomagnetism current law

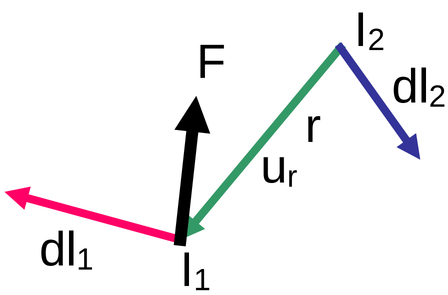
The force exerted on a segment of a masse current I_1 having a vector length of dl_1 and a linear masse density of ρ_1 (kg/m) by a segment of a masse current I_2 having a vector length of dl_2 and a linear masse density of ρ_2; and both separated by a distance r with a unit vector μ_r along r is given by;

$$F = -\frac{G\rho_1 dl_1 \rho_2 dl_2\, u_r}{r^2} + \frac{\mu_g I_1 I_2\, dl_1 \times (dl_2 \times u_r)}{r^2}$$

Unified gravitomagnetism and electromagnetism law

$$F = \frac{(-G\rho_1\rho_2 + k\sigma_1\sigma_2)dl_1 dl_2\, u_r}{r^2} + \frac{(\mu_g I_1 I_2 + \mu_0 I_{e1} I_{e2})dl_1 \times (dl_2 \times u_r)}{r^2}$$

Where σ_1 and σ_2 (A.s/m) are the linear charge densities of electric currents I_{e1} and I_{e2}.

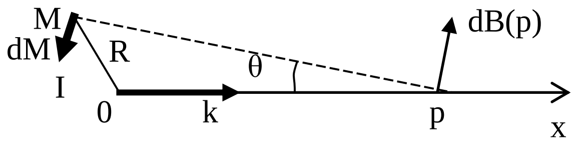


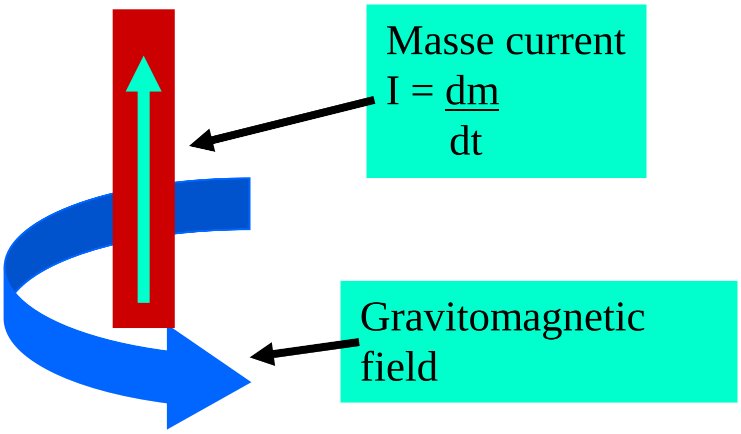
Gravitomagnetism field law

The elementary gravitomagnetic field dB(p) build on the point P by an elementary length of a masse current I, is given by:

$$dB(p) = \frac{\mu_g I\, dM \times u}{4\pi \lVert MP \rVert^2} \quad (s^{-1} \text{ radians par second}),$$

Where u is a unit vector of MP, $u = MP/\lVert MP \rVert$ and μ_g is the vacuum permeability of the gravitomagnetic field.





Lorentz gravitomagnetic force law

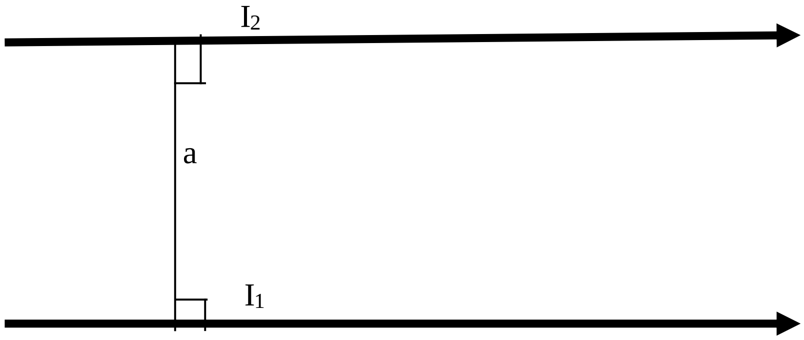
The force to which a particle of masse m_1 and of charge q_1 moving at a velocity V is exerted by a gravitomagnetic field B_g and magnetic field B is given by:

Unified gravitomagnetism and electromagnetism law
$F = V \times (m_1 B_g + q_1 B)$
Lorentz gravitomagnetic and magnetic force

Gravitomagnetic attractive and repulsive forces

The Lorentz gravitomagnetic force is attractive for two parallel line masse currents flowing in the same direction, it is repulsive for two parallel line masse currents flowing in opposite directions. The gravitomagnetic Lorentz force between two parallel line masse currents of infinity length per unit length is given by;

$$\frac{F_y}{dx} = + \frac{\rho_1 . \rho_2}{2\pi\varepsilon_g a} + \frac{\mu_g I_1 I_2}{2\pi a}$$



The Lorentz gravitomagnetic force is attractive for two ring masse currents flowing in the same angular direction, it is repulsive for two ring masse currents flowing in opposite directions. This implies that ring masse currents have north and south poles
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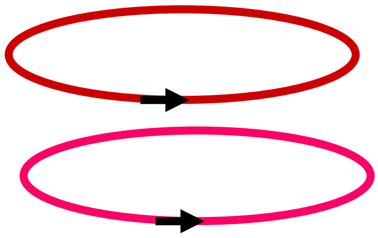
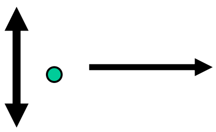
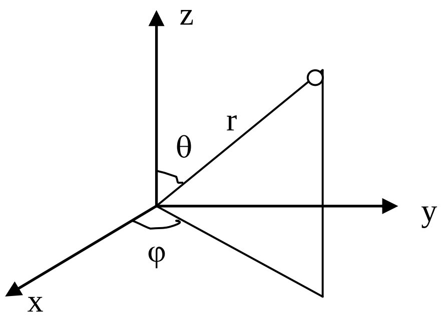


Gravitational waves radiation

When matter vibrates it radiates transversal gravitational waves.



Transversal waves



Let z = a.sin(ωt), where z is the distance of matter masse m from the mean point, a is the amplitude, ω = 2πf and t the time. The velocity v is given by; v = a.ω.co(ωt) (v<<c ➔ a.ω. << c). The masse current ig(t)g is given by;

$$ig(t) = \frac{dm}{dt} = \frac{dm}{dz} \cdot \frac{dz}{dt} = \frac{m \cdot v}{2a} = \frac{m \cdot a.\omega.co(\omega t)}{2a} = \frac{m.\omega.co(\omega t)}{2}$$

If Ig is the maximum current then $I_g = \frac{m.\omega.}{2}$

The average Poynting vector is written as:

$$S = \frac{1}{2\mu_g} R\,[g \times B_g^*] = \frac{\mu_g a^2 (I_g)^2 \omega^2}{8\pi^2 r^2 c} \sin^2(\theta)\ u_r$$

Since $I_g = \frac{m.\omega.}{2}$

Then S is given by;

$$S = \frac{\mu_g m^2 a^2 \omega^4 \sin^2(\theta)}{32\pi^2 r^2 c}\ u_r$$

The average total power radiated P is given by;

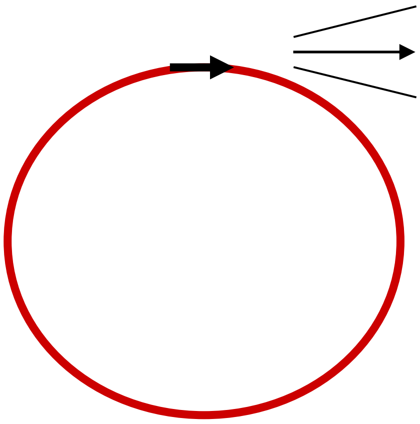
$$\mathbf{P} = \int ds.d\Sigma \text{ , where } \quad d\Sigma = r^2 \sin(\theta) d\varphi\ u_r$$

With $0 < \varphi < 2\pi$ and $0 < \theta < \pi$

$$P = \frac{\mu_g m^2 a^2 \omega^4}{12\pi c}$$

This is the gravitational radiation power lost in space by a vibrating masse particle.

When matter of masse m goes around a curve or accretes a black hole in generates synchrotron gravitational radiation (at relativistic speeds it could generate “infra red to X rays” gravitational waves, at low speeds the frequency could vary from a fraction of a cycle/second up to giga cycles/second). This leads to loss of energy thru gravitational radiation.



Gravitational synchrotron cone beam radiation

If the speed of matter v << c (speed of light), then we can ignore the gravitational synchrotron radiation and just consider the far area dipole radiation (hetz antenna).

The position of a masse particle m around a circular orbit of radius a is given by;

$y = a.\sin(\omega t)$
$x = a.\cos(\omega t)$

This is equivalent to 2 masse particles that execute simple harmonic motions in orthogonal directions x and y.

The average total power radiated P is equal to twice the power of a masse particle that executes a linear simple harmonic motion seen above; thus

$$P = \frac{\mu_g m^2 a^2 \omega^4}{6\pi c}$$

or

$$P = \frac{\mu_g m^2 v^4}{6\pi c a^2}$$

This is the gravitational radiation power lost in space by a masse particle going around a circular orbit.

Gravitomagnetic induction law

The gravitomagnetic induction leads to the production of a potential difference between ends of a masse conductor, which is exposed to a variable gravitomagnetic field. We will call this potential difference, the gravitomotive force.

- The law of Maxwell-Faraday gives the relationship between the induced g.m.f (gravitomotive force) e_g and the gravitomagnetic field B_g flux φ that traverses a masse circuit, it is given by;

$e_g = -\frac{d\varphi}{dt}$ Or $\Phi = \iint_S (B_g ds)$

Local equations

$\mathrm{div}\, g = -\rho/\varepsilon_g$, ρ = masse volume density

$\mathrm{div}\, B_g = 0$

$\mathrm{rot}\, g = -\frac{\partial B_g}{\partial t}$

$\mathrm{rot}\, B_g = \mu_g(j_m + \varepsilon_g \frac{\partial g}{\partial t})$,j_m being the masse current intensity.

With

$\Delta g - \varepsilon_g \mu_g \frac{\partial^2 g}{\partial t^2} = 0$

$\Delta B_g - \varepsilon_g \mu_g \frac{\partial^2 B_g}{\partial t^2} = 0$

The gravitomagnetic waves are propagated at a speed of c in vacuum such that;
$c^2 \varepsilon_g \mu_g = 1$

The gravitomagnetic waves transport energy. The energy local density U is given by:

$U = \varepsilon_g \frac{\|g\|^2}{2} + \frac{\|B_g\|^2}{2\mu_g}$, (kg/m^1s^2)

The energy current is given by the vector of Poynting ∏;

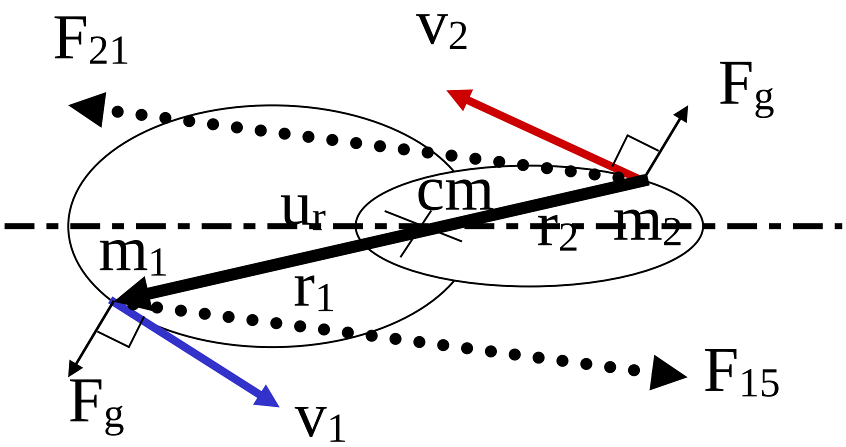
$\prod = \frac{g \times B_g}{\mu_g}$, (kg/s^4)

Two-body problem

Cycle A

F_{21} is the force exerted on masse particle m_2 by m_1

F_{12} is the force exerted on masse particle m_1 by m_2.



The forces between the two masse particles are not radial as describe by the Newton's law. C_m is the center of masse and r is the distance between m_1 and m_2 **r_1 is the distance of m_1 from the center of masse Cm and r_2 is the distance of m_2 from the center of masse C_m. Fg is the gravitomagnetic force exerted on m_1 and m_2**

1) $F_{21} + F_{12} = 0$
2) $m_1r_1 = m_2r_2$, $r = r_1 + r_2$
3) $m_1V_1 + m_2V_2 = 0$
4) $\mathbf{F_g = \{ \mu_g m_1 m_2 V_1 \times (V_2 \times u_r) \} / r^2}$

- The velocities V_1 and V_2 are always in opposite directions.
- The Lorentz gravitomagnetic force $\mathbf{F_g}$ is always perpendicular to the velocities V_1 and V_2

A two-body system is not a closed system, it is an open system that looses energy thru gravitational waves radiation, in that case there is no energy conservation and no angular momentum conservation as stated by the Newton's law, this is also true for an n-body system.

Energy conservation law

The total energy in an open system is constant, thus;

$$E_k + E_p + E_r = \text{constant}$$

Where;
Er = relative kinetic energy of a masse particle
Ep = relative potential energy of a masse particle
Er = gravitomagnetic and electromagnetic radiation energy

As we shall see later, the gravitational (gravitomagnetic) radiation energy from a body going around an orbit in the solar system is minuscule compared to its kinetic energy. It is indeed an extremely good approximation to assume the Newton's law of energy conservation. Thus;

$$E_k + E_p \approx \text{constant}$$

Physical constants

Name	Symbol	Value	Origin
Constant of gravity	G	$\approx 6.674 \times 10^{-11}$ $m^3/kg.s^2$	Measured
Speed of gravitational interaction in vacuum	c	$\approx 2.99 \times 10^8$ m/s	Hypothesis
Vacuum permittivity of gravitational field	ε_g	$\approx 1.18 \times 10^9$ $kg.s^2/m^3$	$\varepsilon_g = 1/4\pi G$
Vacuum permeability of gravitomagnetic field	μ_g	$\approx 9.4 \times 10^{-27}$ m/kg	$c^2\varepsilon_g\mu_g = 1$
Characteristic impedance of gravitomagnetic waves in vacuum	Z_g	$\approx 2.82 \times 10^{-18}$ $m^2/kg.s$	$\mu_g c$
Speed of interaction of electric field	c	$\approx 2.99 \times 10^8$ m/s	Measured
Vacuum permittivity of electric field	ε_0	$4\pi \times 10^{-7}$ $Kg/A^2.s^2$	Definition
Vacuum permeability of magnetic field	μ_0	8.85×10^{-12} $A^2.s^2/kg.m$	$c^2\varepsilon_0\mu_0 = 1$
Characteristic impedance of electromagnetic waves in vacuum	Z_0	376.7 Ω	$\mu_0 c$
Elementary charge	q_e	1.6×10^{-19} A.s	Measured
Masse of the electron	m_e	9.19×10^{-31} kg	Measured
Masse of the proton	m_p	1.672×10^{-27} kg	Measured
Masse of the neutron	m_n	1.674×10^{-27} kg	Measured
Masse of the Earth	Me	5.973×10^{24} kg	Calculated
Masse of the Sun	Ms	1.9891×10^{30} kg	Calculated
Distance between Earth/Sun	r	149 598 000 000m	Calculated

Physical quantities

Nom	symbol	units
Gravitational field	g	m/s^2
Gravitomagnetic field	B_g	s^{-1}
Electric field	E	
Magnetic field	B	

Detailed description of gravitomagnetism

Objective

To show the existence of gravitomagnetic field, we will not propose any theory in this exposé; we will only use mathematical tools that are accepted and experimented by scientists.

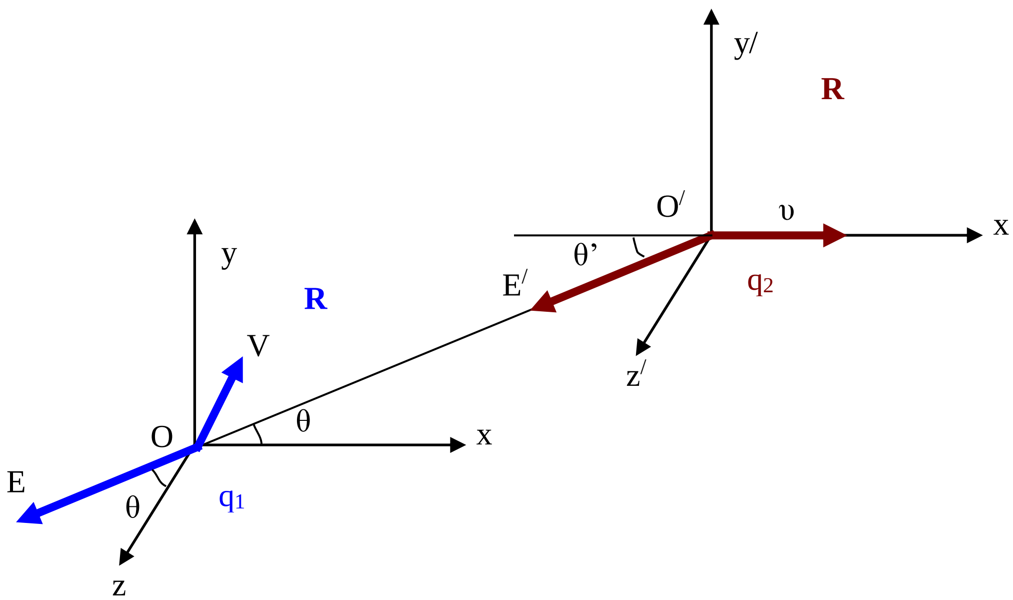
Hypothesis

Most scientists think that the gravitational interaction in vacuum is propagated at a finite speed; this speed is thought to be equal to the speed of light.

Method

In accordance with the reference frame to reference frame forces transformation principle, we will apply the Lorenz transformation to the force exerted by electric field on a particle in order show the existence of magnetic field. Analogically, we will apply the Lorenz transformation to the force exerted by gravitational field on a particle in order to show the existence of gravitomagnetic field. And finally, we will show that light is composed of electromagnetic and gravitomagnetic waves. That wave, thus the light, will be known as electrogravitomagnetic wave.

EXISTENCE OF MAGNETIC FIELD



Consider a reference frame $R^{/}$ in translation along Ox axis at a velocity of υ with respect to a reference frame R. Let q_2 be a charge at rest in the reference frame $R^{/}$, consequently this charge q_2 is also moving at a velocity of υ with respect to the reference frame R. We are going to calculate the force that charge q_2 exerts on the charge q_1, the later q_1 is moving at velocity of V with respect to reference frame R. $E^{/}$ denotes the electric field build by q_2 with respect to reference frame $R^{/}$ at a given radial distance r in space, E denotes the electric field build by q_2 with respect to reference frame R at a given radial distance r in space. The yOx and $y^{/}O^{/}x^{/}$ plains are in the same plain and Ox and $O^{/}x^{/}$ axis are parallel.

Let us find the force exerted by charge q_2 on the charge q_1 with respect to reference frame R. Applying Lorentz transformation to a force permits us to determine how a force is transformed from the reference frame $R^/$ to the reference frame R using the following equations.

$$F_x = \frac{1}{\left(1 + \frac{\beta V^/_x}{c}\right)} \left(F^/_x + \beta \frac{F^/.V^/}{c}\right)$$

$$F_y = \frac{1}{\gamma\left(1 + \frac{\beta V^/_x}{c}\right)} .F^/_y$$

$$F_z = \frac{1}{\gamma\left(1 + \frac{\beta V^/_x}{c}\right)} .F^/_z$$

With ;

$$V^/_x = \frac{(V_x - \upsilon)}{\left(1 - \frac{\beta V_x}{c}\right)}$$

$$V^/_y = \frac{V_y}{\gamma\left(1 - \frac{\beta V_x}{c}\right)}$$

$$V^/_z = \frac{V_z}{\gamma\left(1 - \frac{\beta V_x}{c}\right)}$$

With $\beta = \frac{\upsilon}{c}$ $\quad \gamma = \frac{1}{\sqrt{(1-\beta^2)}}$

Due to aberration, θ is the angle of propagation of the field in the R reference frame and θ' is the angle of propagation of the field in the R' reference frame, with;

$$\text{Cos}(\theta) = \frac{(\cos(\theta') + \beta)}{(1 + \beta\cos(\theta'))}$$

$$\sin(\theta) = \frac{\sin(\theta')}{\gamma.(1 + \beta\cos(\theta'))}$$

$$\tan(\theta) = \frac{\sin(\theta')}{\gamma.(\cos(\theta') + \beta)}$$

Since $v << c$ then we can assume the $\theta \approx \theta'$.

c is the speed of light:

$\upsilon \geq 0$

We deduce that:

$$F_x = q_1E'_x + \frac{\gamma\beta q_1.(E'_yV_y + E'_zV_z)}{c}$$

$$F_y = \gamma q_1E'_y - \frac{\gamma\beta q_1E'_yV_x}{c}$$

$$F_z = \gamma q_1E'_z - \frac{\gamma\beta q_1E'_zV_x}{c}$$

The yOx and y'O'x' plane are in the same plane and Ox and O'x' axis are parallel.

$E'z = 0 \Rightarrow Fz = 0.$

The equations before are simplified down to:

$$F_x = q_1E'_x + \frac{\gamma\beta q_1E'_yV_y}{c}$$

$$F_y = \gamma q_1E'_y - \frac{\gamma\beta q_1E'_yV_x}{c}$$

$$F_z = 0$$

We note the 4 dimensions pseudo norm or quadrivector:

$r^2 = c^2t^2 - \mathbf{r}^2 = r'^2 = c^2t'^2 - \mathbf{r}'^2$, where $\mathbf{r}^2$ et $\mathbf{r}'^2$ are redial vectors that join q1 and q2, r is the distance between the two charges q1 and q2. This quantity $\mathbf{r^2} = c^2t^2 - \mathbf{r}^2 = c^2t'^2 - \mathbf{r}'^2$ does not depend on the reference frame and constitutes an invariant.

In accordance with the invariance of this quantity ;

With $k = 1/4\pi\varepsilon_0$, ε_0 being the vacuum electric field vacuum, then E'_x and E'_y are given by:

$$E'_x = -\frac{kq_2\cos(\theta)}{r^2}$$

$$E'_y = -\frac{kq_2\sin(\theta)}{r^2}$$

$$F_x = -\frac{kq_1q_2\cos(\theta)}{r^2} - \frac{\gamma\beta kq_1q_2\sin(\theta)V_y}{r^2c}$$

$$F_y = -\frac{\gamma kq_1q_2\sin(\theta)}{r^2} + \frac{\gamma\beta kq_1q_2\sin(\theta)V_x}{r^2c}$$

Fundamental equations

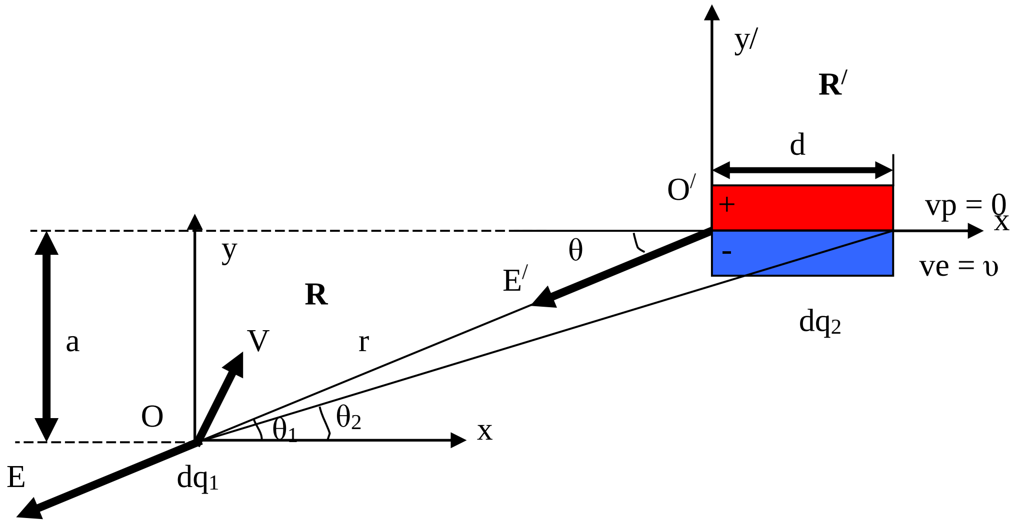
If c → ∞, we get back the Coulomb's law, thus:

$$F_x = -\frac{kq_1q_2\cos(\theta)}{r^2}$$

$$F_y = -\frac{kq_1q_2\sin(\theta)}{r^2}$$

To show the validity of the fundamental equations before, we are going to determine the force exerted, on q_1 moving at a velocity of V with respect to the reference frame R, by a neutral rectilinear conductor parallel to the Ox axis, passed by an electric current of I_2.

This conductor is composed of positive charges that are immobile with respect to the reference frame R and negatives charges moving at a velocity of υ along the Ox axis with respect to the reference frame R, but a rest with the respect to reference frame R/, as shown in the following diagram:



1) We will determine the force due to the positive charge dq_2 on the positive charge dq_1. Since the reference frame R/ associated to the positive charge is at rest with respect to reference frame R, with $dq_2 > 0$, then $\upsilon_p = 0 \Rightarrow \beta = 0$ et $\gamma = 1$
⇨

$$dF_{xp} = -\frac{k.dq_1.dq_2\cos(\theta)}{r^2}$$

$$dF_{yp} = -\frac{k.dq_1.dq_2\sin(\theta)}{r^2}$$

2) We will determine the force due to the negative charge dq_2 on the positive charge dq_1. Since the reference frame R/ associated is moving at a velocity of υ with respect to the reference frame R, $\upsilon_e = \upsilon$ and $dq_2 < 0$:

$$dF_{xe} = +\frac{k.dq_1.dq_2\cos(\theta)}{r^2} + \frac{\gamma\beta k.dq_1.dq_2\sin(\theta)V_y}{r^2 c}$$

$$dF_{ye} = +\frac{\gamma k.dq_1.dq_2\sin(\theta)}{r^2} - \frac{\gamma\beta k.dq_1.dq_2\sin(\theta)V_x}{r^2 c}$$

3) Supposing that the system is linear:

$dF_x = dF_{xp} + dF_{xe}$
And
$dF_y = dF_{yp} + dF_{ye}$

⇨

$$dF_x = 0 + \frac{\gamma\beta k.dq_1.dq_2 \sin(\theta)V_y}{r^2c}$$

$$dF_y = \frac{(\gamma - 1)k.dq_1.dq_2 \sin(\theta)}{r^2} - \frac{\gamma\beta k.dq_1.dq_2 \sin(\theta)V_x}{r^2c}$$

Fundamental equations for a neutral electric conductor

But $\beta = \frac{\upsilon}{c}$,

$$dF_x = + \frac{\gamma.kdq_1.dq_2\, \upsilon\sin(\theta)V_y}{r^2c^2}$$

$$dF_y = + \frac{(\gamma-1)k.dq_1.dq_2 \sin(\theta)}{r^2} - \frac{\gamma.kdq_1.dq_2\, \upsilon\sin(\theta)V_x}{r^2c^2}$$

But

$$\upsilon = \frac{dx}{dt}$$

υ being the velocity of the charge.

$dq_2 .\upsilon = I_2dx$, where I_2 = the electric current

By replacing $dq_2\, \upsilon$ in the two functions dF_x and dF_y, we get ;

$$dF_x = + \frac{\gamma.kdq_1\, I_2dx.\sin(\theta)V_y}{r^2c^2}$$

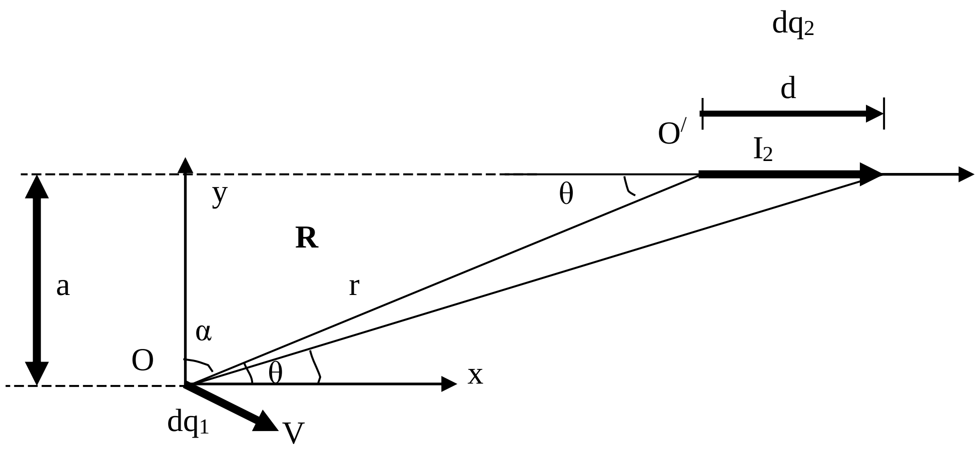
$$dF_y = + \frac{(\gamma-1)\, k.dq_1.dq_2 \sin(\theta)}{r^2} - \frac{\gamma.kdq_1\, I_2dx.\sin(\theta)V_x}{r^2c^2}$$

If $\upsilon << c \Rightarrow \gamma \approx 1$, we get

$$dF_x = + \frac{kdq_1\, I_2dx.\sin(\theta)V_y}{r^2c^2}$$

$$dF_y = - \frac{kdq_1\, I_2dx.\sin(\theta)V_x}{r^2c^2}$$

Changing the variable, if $\theta = \pi/2 - \alpha$, θ varies from π to 0 and α varies from $-\pi/2$ to $\pi/2$, as shown in the following diagram.



$$dF_x = + \frac{kdq_1 I_2 dx.\cos(\alpha)V_y}{r^2c^2}$$

$$dF_y = - \frac{kdq_1 I_2 dx.\cos(\alpha)V_x}{r^2c^2}$$

But

$k = 1/4\pi\varepsilon_0$, ε_0 being the electric field permiattivity, let μ_0 be a variable such that, $\varepsilon_0\mu_0 = 1/c^2$, by replacing c and k in the equations about, we get;

$$dF_x = + \frac{dq_1\mu_0 I_2 dx.\cos(\alpha)V_y}{4\pi r^2}$$

$$dF_y = - \frac{dq_1 \mu_0 I_2 dx.\cos(\alpha)V_x}{4\pi r^2}$$

But we chose the negative charge sense (direction of negative charge drift), it is opposite to the conventional sense (direction of the positive charge drift), we will invert the sign of the velocity υ, thus, invert current I_2 sign in order to be in conformity with the conventional electric current sense.

Where I_2 denotes the conventional current sense, thus opposite to negative charge direction of movement.

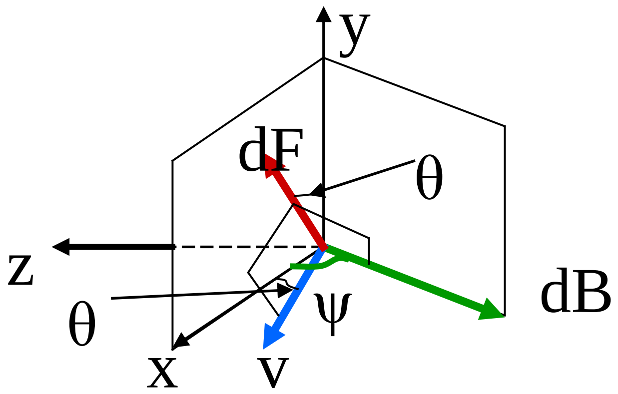
Let,

$$dF_x = - \frac{dq_1\mu_0 I_2 dx.\cos(\alpha)V_y}{4\pi r^2}$$

$$dF_y = + \frac{dq_1 \mu_0 I_2 dx.\cos(\alpha)V_x}{4\pi r^2}$$

We notice that a vector product appears between the velocity V and a field that we shall call B, supposing that a field B exists such that;

$$dF = \begin{pmatrix} dF \\ dFy \\ 0 \end{pmatrix} = qV\times B = q \begin{pmatrix} V \\ Vy \\ 0 \end{pmatrix} \times \begin{pmatrix} dBx \\ dBy \\ dBz \end{pmatrix}$$



ψ is the angle between dB and the xoy plane, V and dB are always perpendicular to dF. Let us determine the angle ψ.

After multiplying up the vector product, we find that;

$$dFx = -\frac{\mu_0 I_2 dx\cos(\alpha).Vy}{4\pi r^2} = -dBy.Vz + dBz.Vy$$

$$dFy = +\frac{\mu_0 I_2 dx\cos(\alpha).Vx}{4\pi r^2} = +dBx.Vz - dBz.Vx$$

$$dFz = 0 = -dBx.Vy + dBy.Vx$$

Vz.dBx = 0, since Vz ≠ 0 ➔ dBx = 0.

Vz.dBy = 0, since Vz ≠ 0 ➔ dBy = 0.

Since (Vx ≠ 0) and (Vy ≠ 0)

➔ $dB_z = -\frac{\mu_0 I_2 dx\cos(\alpha)}{4\pi r^2}$

Consequences;

1) Then angle ψ = π/2, the magnetic field build by an electric current I_2 in the yox plane is perpendicular to that plane; thus

$$dB_z = -\frac{\mu_0 I_2 dx\cos(\alpha)}{4\pi r^2}, \quad dBx = 0 \text{ and } dBy = 0$$

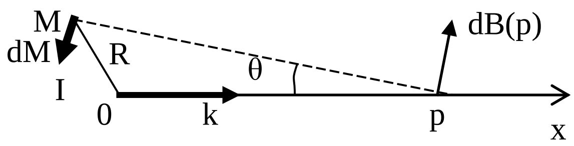
This is known as the Biot and Savart law; this is a fundamental quantitative relationship between an electric current and the magnetic field it produces, based on the experiments done in 1820 by the French scientists Jean-Baptiste Biot and Félix Savart. This is the magnetic equivalent of Coulomb's law, it is the for the magnetic field produced by a short segment of wire, dx, carrying an electric current I_2.

In a more general form;

The elementary magnetic field dB(p) build on the point P by an elementary length of an electric current I, is given by:

$$dB(p) = \frac{\mu_0 \, I \, dM \times u}{4\pi \|MP\|^2}$$

Where u is a unit vector of MP, $u = MP/\|MP\|$ and μ_0 is the vacuum permeability of the magnetic field.



(c)

2) The force exerted on q_1 by a magnetic field B build by the electric current I_2 is given by:

$F = q_1 V \times B$, **this is known as the Lorentz magnetic force**.

We have now shown the existence of the magnetic field by applying the Lorenz transformation to the force exerted on a charged particle.

In that way, by changing the reference frame, the electric field components orthogonal to the movement were slightly modified. The magnetic field is just a relativity effect due to movement and reference frame change.

We deduce that if B is the magnetic field and E_2 the electric field build by a charge q_2 moving at a velocity of υ along the Ox axis, the force to which a charged particle q_1 moving at a velocity of V is exerted is given by:

$F = q_1 E_2 + q_1 V \times B$

The components of that force are:

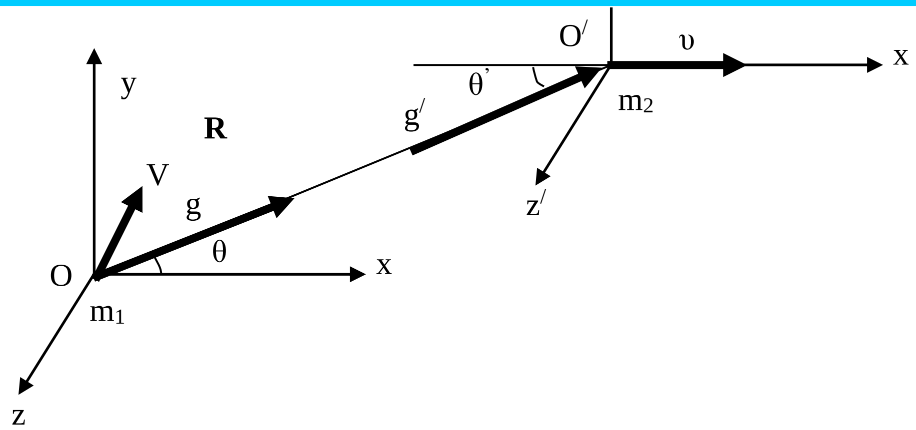
$Fx = q(VyBz - VzBy)$

$Fy = q(VzBx - VxBz)$

$Fz = q(VxBy - VyBx)$

First conclusion

In accordance with the principle of reference frame to reference frame force transformation, by applying the Lorentz transformation to the forced exerted on charged particles by the electric field, we did show the existence of magnetic field. Analogically, by basing our approach on the present scientific knowledge, knowing that the gravity has the same geometrical and propagation properties as the electric field, we shall thereby apply the Lorentz transformation to the force exerted on particles by the gravitational field, in that way we shall show the existence of gravitomagnetic field.



Consider a reference frame $R^/$ in translation along Ox axis at a velocity of υ with respect to a reference frame R. Let m_2 be a masse at rest in the reference frame $R^/$, consequently this masse m_2 is also moving at a velocity of υ with respect to the reference frame R. We are going to calculate the force that masse m_2 exerts on the masse m_1, the later m_1 is moving at velocity of V with respect to the reference frame R. $g^/$ denotes the gravitational field build by m_2 with respect to the reference frame $R^/$ at a given radial distance r in space, g denotes the gravitational field build by m_2 with respect to the reference frame R at a given radial distance r in space. The yOx and $y^/O^/x^/$ plane are in the same plane and Ox and $O^/x^/$ axis are parallel.

Let us find the force exerted by the masse m_2 on the masse m_1 with respect to the reference frame R. Applying Lorenz transformation to a force permits us to determine how a force is transformed from the reference frame $R^/$ to the reference frame R using the following equations:

$$F_x = \frac{1}{\left(1 + \frac{\beta V^/_x}{c}\right)} \left(F^/_x + \beta \frac{F^/.V^/}{c}\right)$$

$$F_y = \frac{1}{\gamma\left(1 + \frac{\beta V^/_x}{c}\right)} .F^/_y$$

$$F_z = \frac{1}{\gamma\left(1 + \frac{\beta V^/_x}{c}\right)} .F^/_z$$

With;

$$V^/_x = \frac{(V_x - \upsilon)}{\left(1- \frac{\beta V_x}{c}\right)}$$

$$V^/_y = \frac{V_y}{\gamma\left(1- \frac{\beta V_x}{c}\right)}$$

$$V^/_z = \frac{V_z}{\gamma\left(1- \frac{\beta V_x}{c}\right)}$$

With $\beta = \frac{\upsilon}{c}$ $\quad \gamma = \frac{1}{\sqrt{(1- \beta^2)}}$

c is the speed of light.

Due to aberration, θ is the angle of propagation of the field in the R reference frame and $\theta^{'}$ is the angle of propagation of the field in the $R^{'}$ reference frame, with;

$$\mathrm{Cos}(\theta) = \frac{(\cos(\theta^{'}) + \beta)}{(1+ \beta\cos(\theta^{'}))}$$

$$\sin(\theta) = \frac{\sin(\theta^{'})}{\gamma.(1+ \beta\cos(\theta^{'}))}$$

$$\tan(\theta) = \frac{\sin(\theta^{'})}{\gamma.(\cos(\theta^{'})+ \beta)}$$

Since v << c then we can assume the $\theta \approx \theta^{'}$.

$\upsilon \geq 0$

We deduce that:

$$F_x = m_1 g'_x + \frac{\gamma\beta m_1.(g'_y V_y + g'_z V_z)}{c}$$

$$F_y = \gamma m_1 g'_y - \frac{\gamma\beta m_1 g'_y V_x}{c}$$

$$F_z = \gamma m_1 g'_z - \frac{\gamma\beta m_1 g'_z V_x}{c}$$

The yOx and $y'O'x'$ plane are in the same plane and Ox and $O'x'$ axis are parallel.

$g'z = 0 \Rightarrow Fz = 0$.

The equations before are simplified down to:

$$F_x = m_1 g'_x + \frac{\gamma\beta m_1 g'_y V_y}{c}$$

$$F_y = \gamma m_1 g'_y - \frac{\gamma\beta m_1 g'_y V_x}{c}$$

$$F_z = 0$$

We note the 4 dimensions pseudo norm or quadrivector:
$r^2 = c^2 t^2 - \mathbf{r}^2 = r'^2 = c^2 t'^2 - \mathbf{r}'^2$ where $\mathbf{r}^2$ and $\mathbf{r}'^2$ are redial vectors that join q1 and q2, r is the distance between the two charges q1 and q2. This quantity $\mathbf{r}^2 = c^2 t^2 - \mathbf{r}^2 = c^2 t'^2 - \mathbf{r}'^2$ does not depend on the reference frame and constitutes an invariant.
In accordance with the invariance of this quantity;
Let $k = G = 1/4\pi\varepsilon_g$, G being the constant of gravity and ε_g being the vacuum permittivity of the gravitational field g'_x and g'_y are given by:

$$g'_x = + \frac{k m_2 \cos(\theta)}{r^2}$$

$$g'_y = + \frac{k m_2 \sin(\theta)}{r^2}$$

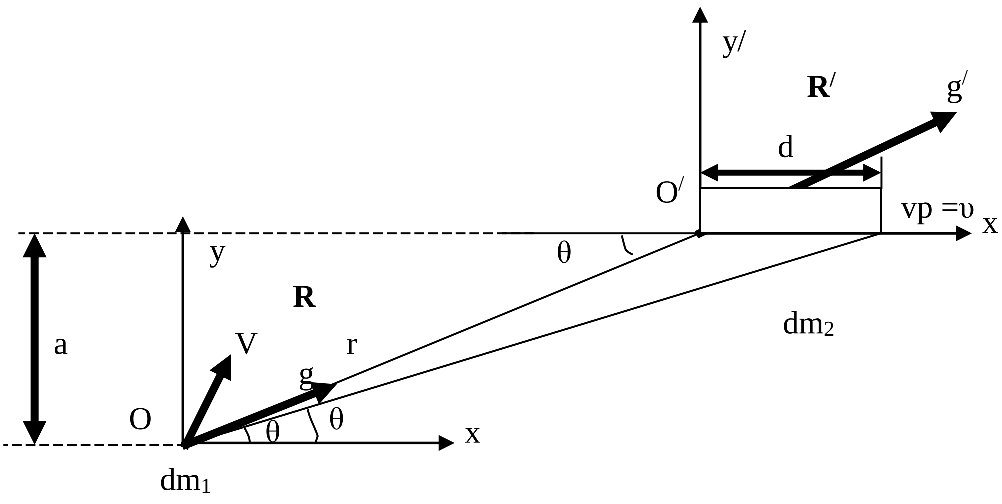
$$F_x = + \frac{k.m_1.m_2 \cos(\theta)}{r^2} + \frac{\gamma\beta k.m_1.m_2 \sin(\theta) V_y}{r^2 c}$$

$$F_y = + \frac{\gamma k.m_1.m_2 \sin(\theta)}{r^2} - \frac{\gamma\beta k.m_1.m_2 \sin(\theta) V_x}{r^2 c}$$

If $c \rightarrow \infty$, we get back Newton's equation, thus:

$$F_x = + \frac{k.m_1.m_2 \cos(\theta)}{r^2}$$

$$F_y = + \frac{k.m_1.m_2 \sin(\theta)}{r^2}$$



Consider a rectilinear pipe masse conductor without masse, of a finite length, parallel to the vector $O^{/}x^{/}$, passed by a constant masse current of I_2 and dx a given point elementary displacement along the $O^{/}x^{/}$. We will determine the force due to the masse dm_2 on dm_1. Since the reference frame $R^{/}$ associated is moving at a velocity of υ with respect to reference frame R, dF_x and dF_y are given by:

$$dF_x = + \frac{k.dm_1.dm_2 \cos(\theta)}{r^2} + \frac{\gamma\beta k.dm_1.dm_2 \sin(\theta)V_y}{r^2 c}$$

$$dF_y = + \frac{\gamma k.dm_1.dm_2 \sin(\theta)}{r^2} - \frac{\gamma\beta k.dm_1.dm_2 \sin(\theta)V_x}{r^2 c}$$

But $\beta = \upsilon/c$:

$$dF_x = + \frac{k.dm_1.dm_2 \cos(\theta)}{r^2} + \frac{\gamma dm_1.dm_2\, \upsilon \sin(\theta)V_y}{r^2 c^2}$$

$$dF_y = + \frac{\gamma k.dm_1.dm_2 \sin(\theta)}{r^2} - \frac{\gamma dm_1.dm_2\, \upsilon \sin(\theta)V_x}{r^2 c^2}$$

Since two masses attract each other, then they behave like two charges of opposite signs, just like opposite electric currents of opposite senses, we will invert the sign of the velocity υ, in order to be in conformity with the electric current convention law.

Then dF_x and dF_y are given by:

$$dF_x = + \frac{k.dm_1.dm_2 \cos(\theta)}{r^2} - \frac{\gamma k dm_1.dm_2\, \upsilon \sin(\theta)V_y}{r^2 c^2}$$

$$dF_y = + \frac{\gamma k.dm_1.dm_2 \sin(\theta)}{r^2} + \frac{\gamma k dm_1.dm_2\, \upsilon \sin(\theta)V_x}{r^2 c^2}$$

But:

$\upsilon = \frac{dx}{dt}\upsilon$, is the velocity of the masse.

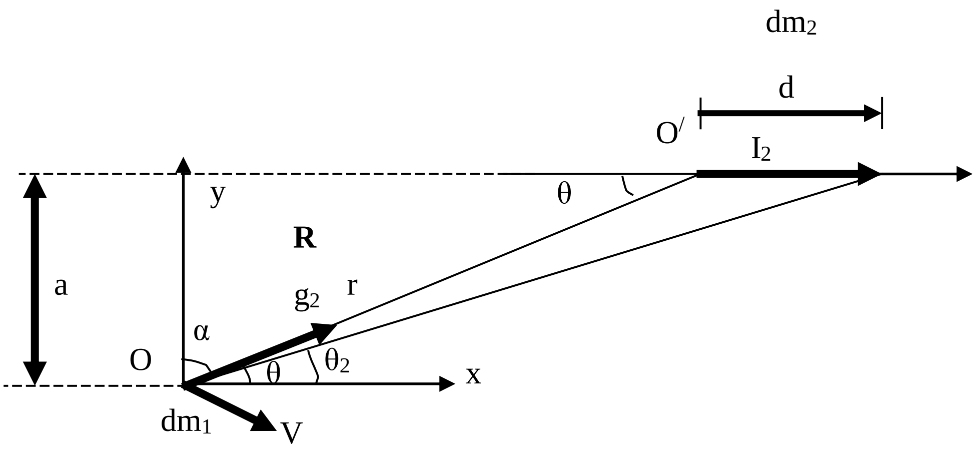
$dm_2\, \upsilon = I_2 dx$, where I_2 = masse current.

By replacing $dm_2\,\upsilon$ in the functions dF_x and dF_y, we get:

$$dF_x = +\frac{k.dm_1.dm_2\cos(\theta)}{r^2} - \frac{\gamma dm_1\, I_2 dx.\sin(\theta)V_y}{r^2c^2}$$

$$dF_y = +\frac{\gamma k.dm_1.dm_2\sin(\theta)}{r^2} + \frac{\gamma dm_1\, I_2 dx.\sin(\theta)V_x}{r^2c^2}$$

Changing the variable, if $\theta = \pi/2 - \alpha$, then θ varies from π to 0 and α varies from $-\pi/2$ to $\pi/2$, as shown in the following diagram:



$$dF_x = +\frac{k.dm_1.dm_2\sin(\alpha)}{r^2} - \frac{\gamma k\, dm_1\, I_2 dx.\cos(\alpha)V_y}{r^2c^2}$$

$$dF_y = +\frac{\gamma\, k.dm_1.dm_2\cos(\alpha)}{r^2} + \frac{\gamma k\, dm_1\, I_2 dx.\cos(\alpha)V_x}{r^2c^2}$$

But

$k = G = 1/4\pi\varepsilon_g$, ε_g being the gravitational field permittivity, let μ_g be a variable such that variable; $\varepsilon_g\mu_g = 1/c^2$, by replacing c and k in the equations about, we get:

$$dF_x = +\frac{dm_1.dm_2\sin(\alpha)}{4\pi\varepsilon_g r^2} - \frac{\gamma\, dm_1\mu_g\, I_2 dx.\cos(\alpha)V_y}{4\pi\, r^2}$$

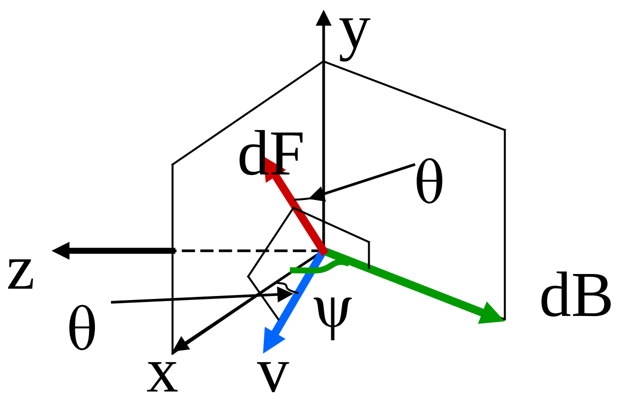
$$dF_y = +\frac{\gamma\, dm_1.dm_2\cos(\alpha)}{4\pi\varepsilon_g r^2} + \frac{\gamma\, dm_1\,\mu_g\, I_2 dx.\cos(\alpha)V_x}{4\pi\, r^2}$$

We notice that a vector product appears between the velocity V and a field the we shall call B_g, supposing a field B_g exists such that:

$$dF2 = \begin{pmatrix} dFx \\ dFy2 \\ 0 \end{pmatrix} = mV\times B_g = m \begin{pmatrix} V \\ Vy \\ 0 \end{pmatrix} \times \begin{pmatrix} dBgx \\ dBgy \\ dBgz \end{pmatrix}$$

$$dF_{x2} = - \frac{\gamma\, dm_1 \mu_g\, I_2 dx.\cos(\alpha) V_y}{4\pi\, r^2}$$

$$dF_{y2} = + \frac{\gamma\, dm_1\, \mu_g\, I_2 dx.\cos(\alpha) V_x}{4\pi\, r^2}$$



ψ is the angle between dBg and the xoy plane, V and dB are always perpendicular to dF. Let us determine the angle ψ.

After multiplying up the vector product, we find that;

$$dFx2 = - \frac{\mu_g\, I_2 dx \cos(\alpha).\ Vy}{4\pi\, r^2} = -dBgy.Vz + dBgz.Vy$$

$$dFy2 = + \frac{\mu_g\, I_2 dx \cos(\alpha).\ Vx}{4\pi\, r^2} = +dBgx.Vz - dBgz.Vx$$

$$dFz2 = 0 = - dBgx.Vy + dBgy.Vx$$

Vz.dBgx = 0, since $Vz \neq 0$ ➔ dBgx = 0.

Vz.dBgy = 0, since $Vz \neq 0$ ➔ dBgy = 0.

Since ($Vx \neq 0$) and ($Vy \neq 0$)

➔ $$dBg_z = - \frac{\mu_g\, I_2 dx \cos(\alpha)}{4\pi\, r^2}$$

Consequences;

1) Then angle $\psi = \pi/2$, the gravitomagnetic field build by a masse current I_2 in the yox plane is perpendicular to that plane; thus

$$dBg_z = -\frac{\mu_g I_2 dx \cos(\alpha)}{4\pi r^2}, \quad dBx = 0 \text{ and } dBy = 0$$

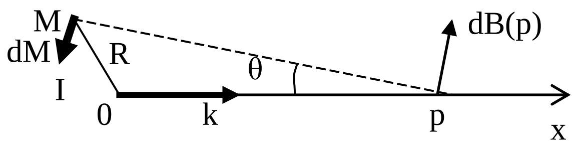
This is known as the Biot and Savart law; this is a fundamental quantitative relationship between a masse current and the gravitomagnetic field it produces. This is the gravitomagnetic equivalent of Newton's law, it is the for the gravitomagnetic field produced by a short segment of pipe, dx, carrying a masse current I_2.

In a more general form;

The elementary gravitomagnetic field dB(p) build on the point P by an elementary length of a masse current I, is given by:

$$dB(p) = \frac{\mu_g I\, dM \times u}{4\pi \|MP\|}$$

Where u is a unit vector of MP, $u = MP/\|MP\|$ and μ_g is the vacuum permeability of the gravitomagnetic field.



(c)

2) The force exerted on q_1 the gravitomagnetic field Bg build by the masse current I_2 is given by:

$F = m_1 V \times Bg$, **this is known as the Lorentz gravitomagnetic force**.

We have now shown the existence of the gravitomagnetic field by applying the Lorenz transformation to the force exerted on a masse particle, μ_g being the vacuum permeability of gravitomagnetic field.

In that way, by changing the reference frame, the gravitational field components orthogonal to the movement were slightly modified. The gravitomagnetic field is just a relativity effect due to movement and reference frame change.

We deduce that if B_g is the gravitomagnetic field and g_2 the gravitational field build by a masse m_2 moving at a velocity of υ along the Ox axis, the force to which a masse particle m_1 moving at a velocity of V is exerted is given by:

$F = m_1 g_2 + m_1 V \times B_g$
with

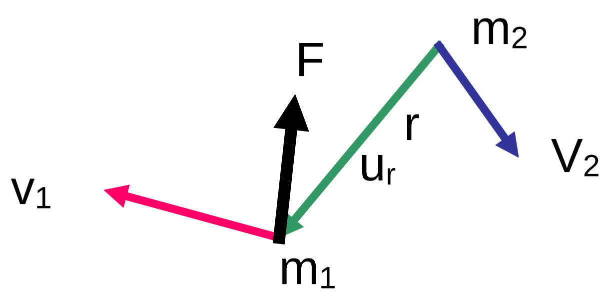
The components of that force are :

$Fx = q(VyB_gz - VzB_gy)$
$Fy = q(VzB_gx - VxB_gz)$
$Fx = q(VxB_gy - VyB_gx)$

Fundamental equations

If the gravitomagnetic field is generated by a moving particle then we can deduce the force between two particles, thereby deduce the new **Newton's law**.

New Newton's law



The force exerted on a particle of masse m_1 moving at a velocity of V_1 by a particle of masse m_2 moving at a velocity of V_2 and both separated by a distance r with a unit vector u_r along r is given by;

$$F = -\frac{Gm_1m_2}{r^2}u_r + \frac{\mu_g m_1 m_2 V_1\times(V_2\times u_r)}{r^2}$$

Where G is the constant of gravity and μ_g is the vacuum permeability of gravitomagnetic field. **We notice that the force F is not radial as described by Newton's law of gravity.** $V_1\times(V_2\times u_r)$ is a vector product, the $(V_2\times u_r)$ vector product must be done first because the triple vector product is not associative.

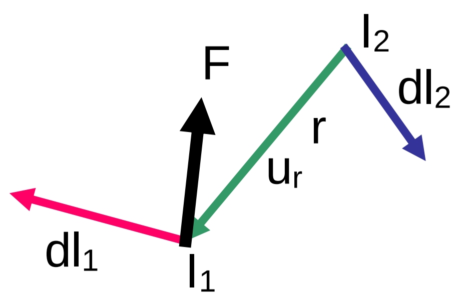
If the particles are charged the force exerted on a particle of masse m_1 and of a charge of q_1 moving at a velocity of V_1 by a particle of masse m_2 and of a charge of q_2 moving at a velocity of V_2 is given by;

Unified gravitomagnetism and electromagnetism law

$$F = \left(-\frac{Gm_1m_2 + kq_1q_2}{r^2}\right)u_r + \left(\frac{\mu_g m_1m_2 + \mu_0 q_1q_2}{r^2}\right)V_1\times(V_2\times u_r)$$

Where $k = 1/4\pi\varepsilon_0$, ε_0 is the vacuum permittivity of the electric field and μ_0 is the vacuum permeability of magnetic field.

Gravitomagnetism current law



The force exerted on a segment of a masse current I_1 having a vector length of dl_1 and a linear masse density of ρ_1 by a segment of a masse current I_2 having a vector length of dl_2 and a linear masse density of ρ_2; and both separated by a distance r with a unit vector μ_r along r is given by;

$$F = -\frac{G\rho_1 dl_1 \rho_2 dl_2}{r^2}u_r + \frac{\mu_g I_1 I_2\, dl_1\times(dl_2\times u_r)}{r^2}$$

Unified gravitomagnetism and electromagnetism law

$$F = \left(-\frac{G\rho_1\rho_2 + k\sigma_1\sigma_2)dl_1dl_2}{r^2}\right)u_r + \left(\frac{\mu_g I_1I_2 + \mu_0 I_{e1}I_{e2}}{r^2}\right)dl_1\times(dl_2\times u_r)$$

Where σ_1 and σ_2 are the linear charge densities of electric currents I_{e1} and I_{e2}.

Let:

$$dF_x = + \frac{dm_1.dm_2 \sin(\alpha)}{4\pi\varepsilon_g\, r^2} - \frac{\gamma\, dm_1 \mu_g\, I_2 dx.\cos(\alpha) V_y}{4\pi\, r^2}$$

$$dF_y = + \frac{\gamma\, dm_1.dm_2 \cos(\alpha)}{4\pi\varepsilon_g\, r^2} + \frac{\gamma\, dm_1\, \mu_g\, I_2 dx.\cos(\alpha) V_x}{4\pi\, r^2}$$

We will replace dx and r^2 in the equations before to have α as the only variable, ρ_2 being the masse linear density, we get:

$$x = a.\tan(\alpha)$$

$$dm_2 = \rho_2 dx$$

$$dx = \frac{a.d\alpha}{\cos^2(\alpha)}$$

$$r^2 = \frac{a^2}{\cos^2(\alpha)}$$

$$dF_x = + \frac{dm_1.\rho_2 \sin(\alpha) d\alpha}{4\pi\varepsilon_g\, a} - \frac{\gamma\, dm_1 \mu_g\, I_2.\cos(\alpha) V_y d\alpha}{4\pi\, a}$$

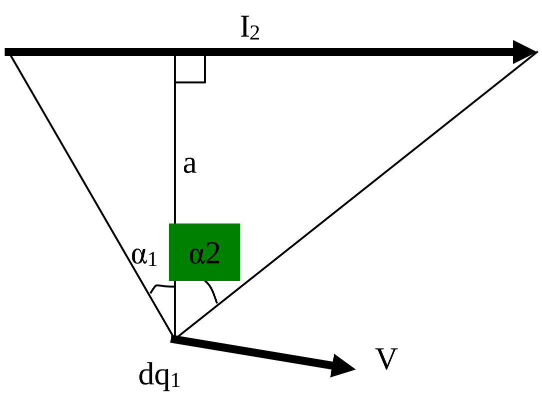
$$dF_y = + \frac{\gamma\, dm_1.\rho_2 \cos(\alpha) d\alpha}{4\pi\varepsilon_g\, a} + \frac{\gamma\, dm_1\, \mu_g\, I_2.\cos(\alpha) V_x d\alpha}{4\pi\, a}$$

By integration from α_1 to α_2 where α varies from $-\pi/2$ to $\pi/2$, we find;

$$dF_x = - \frac{dm_1.\rho_2\, [\cos(\alpha_2) - \cos(\alpha_1)]}{4\pi\varepsilon_g\, a} - \frac{\gamma\, dm_1 \mu_g\, I_2.V_y[\sin(\alpha_2) - \sin(\alpha_1)]}{4\pi\, a}$$

$$dF_y = + \frac{\gamma\, dm_1.\rho_2\, [\sin(\alpha_2) - \sin(\alpha_1)]}{4\pi\varepsilon_g\, a} + \frac{\gamma\, dm_1\, \mu_g\, I_2.V_x[\sin(\alpha_2) - \sin(\alpha_1)]}{4\pi\, a}$$

Schematically we get:



Masse current of infinite length

Implies that, $\alpha_1 = -\pi/2$ and $\alpha_2 = \pi/2$, then F_x and F_y are given by:

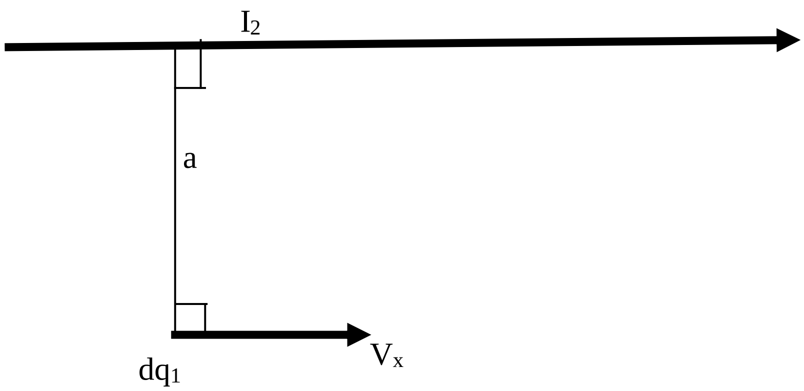
$$F_x = +0 - \frac{\gamma.\mu_g dm_1 I_2 . V_y}{2\pi a}$$

$$F_y = +\frac{\gamma.dm_1 . \rho_2}{2\pi\varepsilon_g a} + \frac{\gamma.\mu_g dm_1 I_2 .V_x}{2\pi a}$$

If the masse dm_1 is moving along the Ox axis, then Vy = 0, as shown in the following diagram:

$$F_x = 0$$

$$F_y = +\frac{\gamma.dm_1 . \rho_2}{2\pi\varepsilon_g a} + \frac{\gamma.\mu_g dm_1 I_2 .V_x}{2\pi a}$$



If I_1 is the masse current build by a particle having a masse dm_1 and ρ_1 is the masse linear density, we get: $dm_1.Vx = I_1dx$, where I_1 = masse current

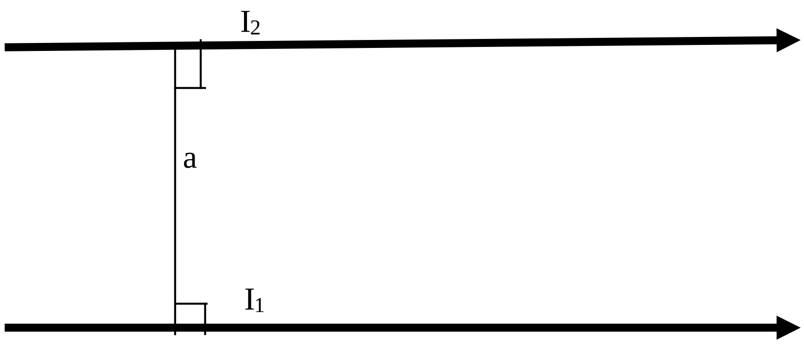
$$dm_1 = \rho_1 dx$$

We get:

$$F_y = +\frac{\gamma.\rho_1 . \rho_2\, dx}{2\pi\varepsilon_g a} + \frac{\gamma.\mu_g I_1 I_2 .dx}{2\pi a}$$

$$\frac{F_y}{dx} = +\frac{\gamma.\rho_1 . \rho_2}{2\pi\varepsilon_g a} + \frac{\gamma.\mu_g I_1 I_2}{2\pi a}$$

Thus, the force par unit length between two parallel masse pipes passed by two masse currents I_1 and I_2 in the same direction.



<u>Attractive Lorentz gravitomagnetic force</u>

$$\frac{F_y}{dx} = + \frac{\gamma.\rho_{1}.\,\rho_2}{2\pi\varepsilon_g\, a} + \frac{\gamma.\mu_g I_1\, I_2}{2\pi a}$$

Observation; when two parallel masse pipes without masse are passed by two masse currents I_1 and I_2 in the same direction, the Lorentz gravitomagnetic force is attractive

<u>Repulsive Lorentz gravitomagnetic force</u>

Observation; when two parallel masse pipes without masse are passed by two masse currents I_1 and I_2 in opposite directions, the Lorentz gravitomagnetic force is repulsive.

$$\frac{F_y}{dx} = 0 \Rightarrow + \frac{\gamma.\rho_{1}.\,\rho_2}{2\pi\varepsilon_g\, a} - \frac{\gamma.\mu_g I_1\, I_2}{2\pi a} = 0$$

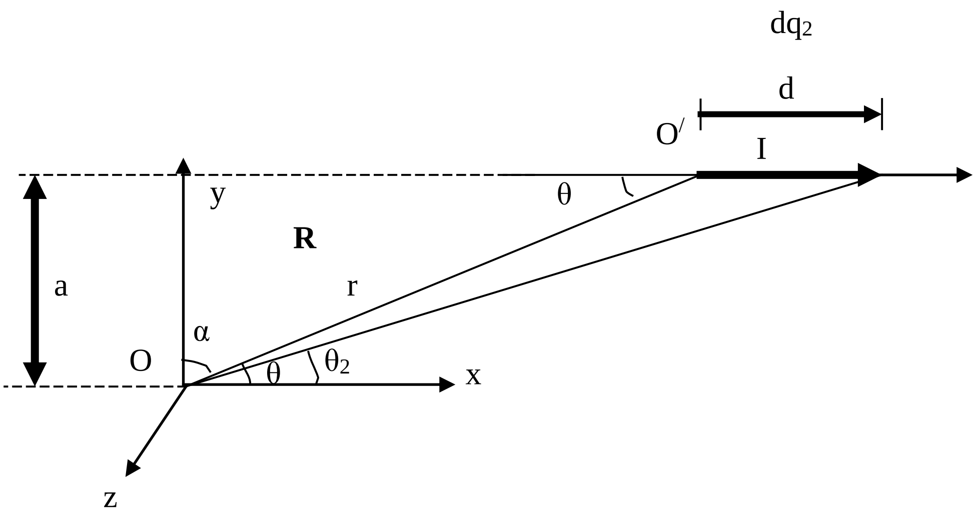
<u>Remark 1:</u> If we put aside gravitational force, two masses moving in the same direction along two parallel lines attract each other and two masses moving in the opposite directions along two parallel lines repulse each other due to the gravitomagnetic field.

<u>Second conclusion:</u> In accordance with the principle of reference frame to reference frame force transformation, by applying the Lorentz transformation to the force exerted by gravitational field on particles, we did show the existence of gravitomagnetic field.

Coupling coefficient between the magnetic and the gravitomagnetic field

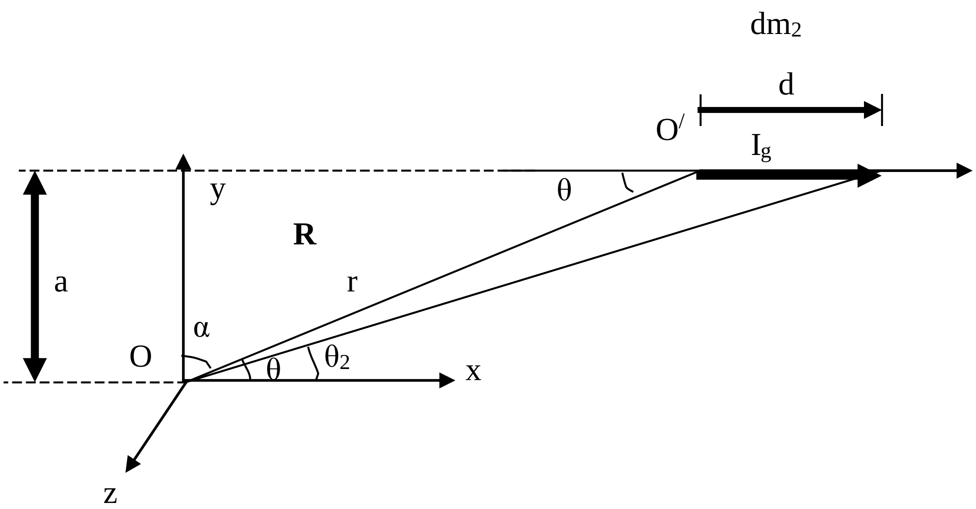
We have already shown that the magnetic field B is given by;

$$dB = -\frac{\mu_0 \, I \, dx \cos(\alpha)}{4\pi \, r^2}$$



We have already shown that the gravitomagnetic field B_g is given by;

$$dB_g = -\frac{\mu_g \, I_g dx \cos(\alpha)}{4\pi \, r^2}$$



We are going to express the masse current I_g as function of the electric current I, the electric current is build up of mobile electrons of masse m_e et charge q_e, in that case an electric current builds up a masse current due to the drift of electrons having masse m_e. The coordinates of the masse of an electron are the same as that of the charge. Let:

- q_e be the charge of the electron
- m_e be the masse of the electron
- n be the number of the electrons
- I be the electric current
- I_g be the masse current
- Q be the total electric charge
- t denotes the time

$I = dQ/dt$

$I_g = dm/dt$

The number of electrons n is given by:
$n = Q/q_e$

The total masse m of electrons on drift is given by:
$m = n.\ m_e$

$$I_g = \frac{dm}{dt} = m_e \frac{dn}{dt} = \frac{m_e\, dQ}{q_e.\, dt} = \frac{m_e}{q_e} . I$$

The charge of an electron is a negative, scalar quantity; in that case I_g and I are opposite directions.
But

$$dB_g = -\frac{\mu_g I_g dx \cos(\alpha)}{4\pi\, r^2}$$

By substituting I_g in the equation before, we get:

$$dB_g = -\frac{m_e.}{q_e}\frac{\mu_g I\, dx \cos(\alpha)}{4\pi\, r^2}$$

The magnetic force B is given by:

$$dB = -\frac{\mu_0 I\, dx \cos(\alpha)}{4\pi\, r^2}$$

By dividing dB_z by dB_{gz}, we get:

$$dB = \frac{q_e . \mu_0}{m_e\, \mu_g} . dB_g$$

By integration, we find:

$$B = \frac{q_e \, . \, \mu_0}{m_e \, \mu_g} \, . \, B_g + (\text{constant} = 0)$$

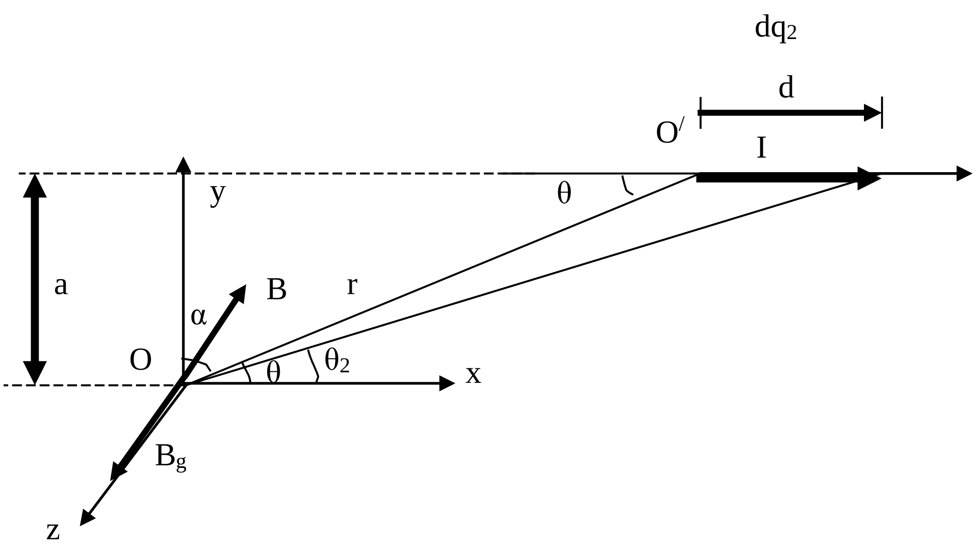
$$B_g = \frac{m_e \, \mu_g}{q_e \, \mu_0} \, . \, B$$

$B_g = \frac{m_e \, \mu_g}{q_e \, \mu_0} \, . \, B = \psi.B$, ψ is the coupling coefficient of an electron

$\psi = 6.1\times10^{-27}\ kg.m^2/A^3.s^3$

The constant of integration is equal to zero, because zero electric current implies zero masse current and vice versa.

The diagram below shows the magnetic field B and the gravitomagnetic field B_g in an electric circuit where the electrons are carriers of the charge and the masse, if protons were the charge and masse carriers then the gravitomagnetic field would be 1600 times stronger because the proton is roughly 1600 times heavier than the electron.



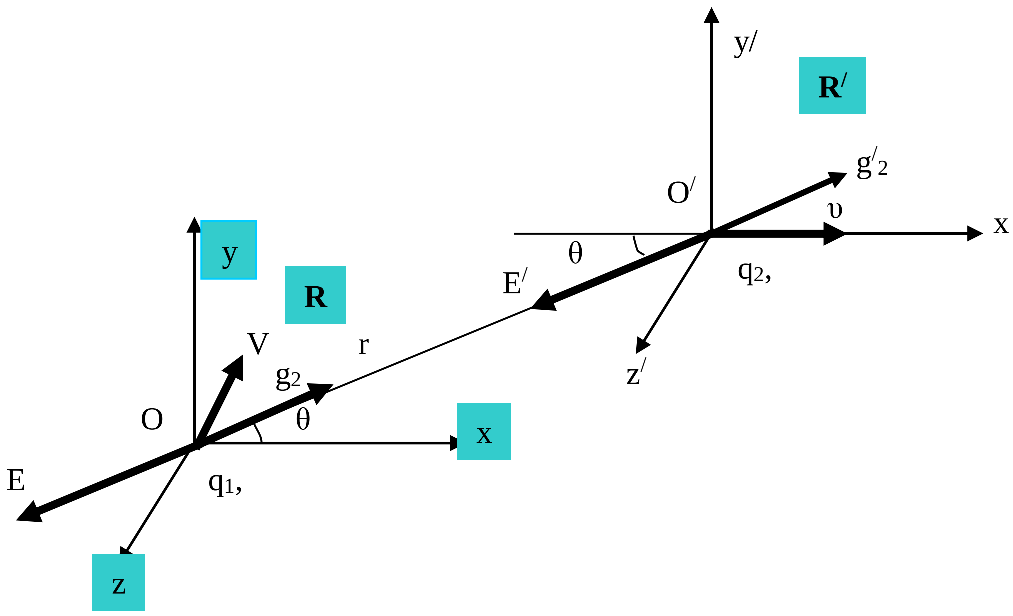
We notice that an electric current builds up a masse current due to the drifting of the masse of the electrons. This is very important because we will later see that the light is not only made up of electromagnetic waves but it is also made up gravitomagnetic waves. We will call this wave, **<u>the electrogravitomagnetic wave</u>**.

Masse and charged particle unified law

We deduce that if B is the magnetic field, E_2 the electric field, B_g the gravitomagnetic field and g_2 the gravitational field all build by a particle of charge q_2 and of masse m_2 moving at a velocity of υ along the Ox axis, the force to which a particle of charge q_1 and of masse m_1 moving at a velocity V is exerted is given by:

$$F = q_1E_2 + m_1g_2 + V\times(q_1B + m_1B_g)$$

Fundamental electrogravitomagnetic equation



Third Conclusion

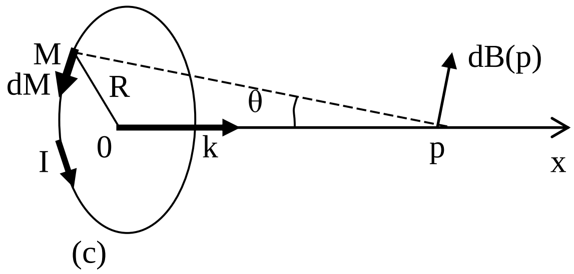
In accordance with the principle of reference frame to reference frame force transformation, by applying the Lorenz transformation to the force exerted by the gravitational field on particles, we have shown the existence of gravitomagnetic field.

Remark 2: Since the Lorenz transformation to the force exerted by the gravitational field on particles gives exactly the same results as the law of Biot and Savart applied to a constant rectilinear masse current, and if we consider a constant masse current given by any graph as a succession of infinitely small constant masse currents, by integration we can calculate on a given point the gravitomagnetic field by using the law of Biot and Savart as a mathematical tool.

Consequences:

Ring masse current

Consider a circular ring C having a centre 0 and a radius R, passed by a masse current I (ring in rotation). The 0x axis is parallel to the unit vector k as shown in the diagram below. Let us determine the gravitomagnetic field using the law of Biot and Savart as a mathematical tool.



M is a point on the ring, in accordance with the law of Biot and Savart, the elementary gravitomagnetic field dB(p) build on the point P on x axis by an elementary length of the ring, and in the same direction as the current, is given by:

$$dB(p) = \frac{\mu_g I \, dM \times u}{4\pi \|MP\|^2}$$

Where u is a unit vector of MP, $u = MP/\|MP\|$.

Two elementary symetrical currents with respect to 0, build at point P, on the plain y0z, two gravitomagnetic fields of opposite signs but equal in magnitude, these fields null each other. We will thereby just have to determine the projection of dB(p) on the 0x axis.

θ denotes the angle 0pM. The angle between dB(p) and k is then $\pi/2 - \theta$ and the projection of dB(p) on the 0x axis is given by:

$$dB(p)_x = \frac{\mu_g I \, dM \sin(\theta)}{4\pi \|MP\|^2}$$

By integration on the whole length of the ring, we get:

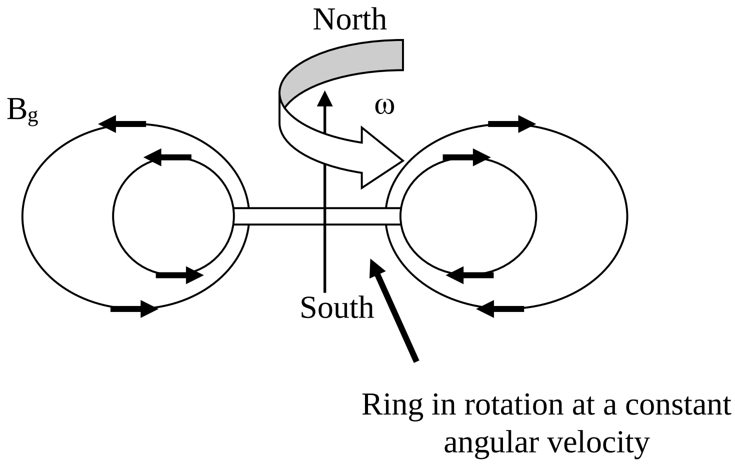
$$B(p)_x = \frac{\mu_g I \, 2\pi R \sin(\theta)}{4\pi \|MP\|^2}$$

$$\|MP\| = R/\sin(\theta)$$

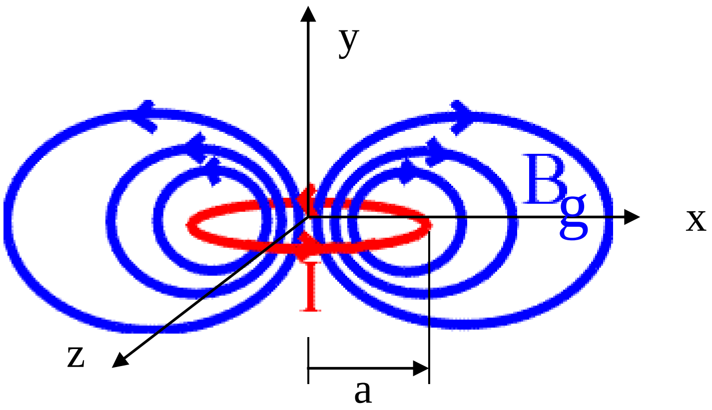
⇨ The gravitomagnetic field is given by the following expression:

$$B(p)_x = \frac{\mu_g I \sin^3(\theta)}{2R}$$

The following diagram shows the gravitomagnetic field around the ring;



The gravitomagnetic field around a ring masse current I



In the xoy

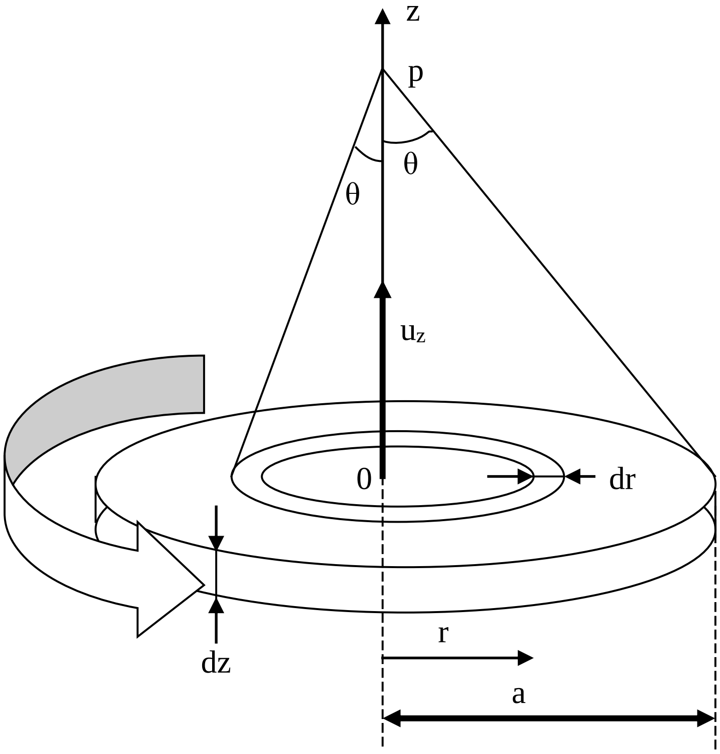
$$B_x = \frac{\mu_g I a}{2\pi}\int_0^{\pi}\frac{y\cos(\theta)\,d\theta}{(x^2 + y^2 + a^2 - 2ax\cos(\theta))^{3/2}}$$

$$B_y = \frac{\mu_g I a}{2\pi}\int_0^{\pi}\frac{(a - x\cos(\theta))\,d\theta}{(x^2 + y^2 + a^2 - 2ax\cos(\theta))^{3/2}}$$

(Bessel integral)

$$B_z = 0$$

Disc masse current



Let σ be the volume density.

A disc in rotation around the 0z axis is equivalent to a succession of circular rings centred at 0, having a radius of r, a width of dr, a depth of dz and passed by masse current of I. Each ring builds at the point p, along the 0z axis, an elementary gravitomagnetic field dB(p,dz), ω = 2πf, f being the frequency of rotation:

$$dB(p,dz)_z = \mu_g \frac{I \sin^3(\theta)}{2r}$$

I = dm/dt is the quantity of masse that traverses (crosses) the surface dr.dz per unit of time.

I = dm/dt = σ.drdz.speed = σ.drdz.rω

$$dB(p,dz)_z = \frac{\mu_g\ \sigma.drdz.r\omega\ \sin^3(\theta)}{2r}$$

$$dB(p,dz)_z = \frac{\mu_g\ \sigma.\omega drdz.\ \sin^3(\theta)}{2}$$

By differentiating the equation tan(θ) = r/z, we find $dr = zd\theta/\cos^2(\theta)$, We thereby get the expression of the elementary gravitomagnetic field as a function of θ:

$$dB(p,dz)_z = \frac{\mu_g\ \sigma.\omega zdz.\sin^3(\theta)d\theta}{2\cos^2(\theta)}$$

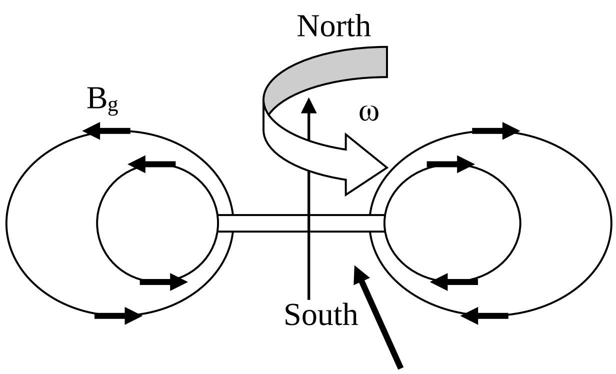
The total gravitomagnetic field build by a rotating disc becomes, taking the extreme angle θ_0 such that $\tan(\theta_0) = a/z$ and $\cos(\theta_0) = z/\sqrt{(a^2 + z^2)}$:

$$\frac{\sin^3(\theta)}{\cos^2(\theta)} = \frac{\sin^2(\theta)\ \sin(\theta)}{\cos^2(\theta)} = \frac{(1-\cos^2(\theta))\sin(\theta)}{\cos^2(\theta)} = \frac{\sin(\theta)}{\cos^2(\theta)} - \sin(\theta)$$

By integration from 0 to θ_0;

$$dB(p,dz)_z = \frac{\mu_g\ \sigma.\omega z dz.}{2}\left(\frac{1}{\cos(\theta_0)} + \cos(\theta_0) - 2\right)$$

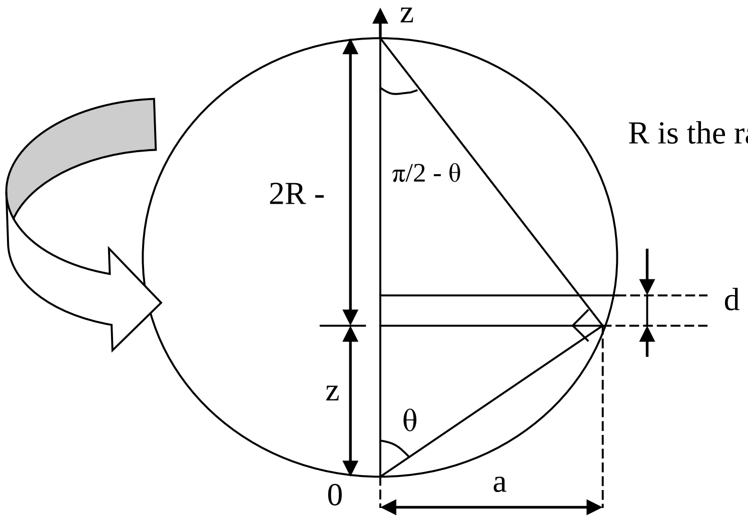
The following diagram shows the schematic form of the gravitomagnetic field of a disc in rotation at a constant angular velocity:



Masse disc in rotation at a constant rate

<u>Remark 3</u>: If we put aside gravitational force, consequence of remark 1, two rings/discs in rotation at a constant angular velocity in the same axis and in the same angular direction, attract each other. Two rings/discs in rotation at a constant angular velocity in the same axis and in opposite angular directions, repulse each other. Masses in rotation can thereby be considered as having polarities, north and south just like electromagnets.

Full sphere masse current



R is the radius of the sphere

A sphere in rotation is equivalent to a succession of discs having 0z axis as the centre, of a radius a, of thickness dz and of density σ. Each disc builds on the point 0 along the 0z axis the elementary gravitomagnetic field, $\omega = 2\pi f$, f being the frequency of rotation. **Take care**; the vacuum permeability μ_g of gravitomagnetic field in matter is not perhaps equal to that one in vacuum, the same case regarding the gravitational interaction speed.

$$dB(0)_z = \frac{\mu_g\ \sigma.\omega z dz.}{2}\left(\frac{1}{\cos(\theta)} + \cos(\theta) - 2\right)$$

We will determine zdz:

$$\frac{a}{z} = \tan(\theta) \qquad \text{and} \qquad \frac{a}{(2R-2)} = \tan(\pi/2 - \theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

➔ $z = 2R\cos^2(\theta)$

$dz = -\ 4R\sin(\theta)\cos(\theta)d\theta$

$zdz = -\ 8R^2 \sin(\theta)\cos^3(\theta)d\theta$

By replacing zdz in function $dB(0)_z$, we get:

$$dB(0)_z = -\ 4\mu_g\ \sigma.\omega R^2\ [\sin(\theta)\cos^2(\theta)+\sin(\theta)\cos^4(\theta)-2\sin(\theta)\cos^3(\theta)]d\theta$$

By integration from π/2 to 0:

$$B(0)_z = -4\mu_g\,\sigma.\omega R^2 \left[-\frac{\cos^3(\theta)}{3} - \frac{\cos^5(\theta)}{5} + \frac{\cos^4(\theta)}{2}\right]_{\pi/2}^{0}$$

$$B(0)_z = -4\mu_g\,\sigma.\omega R^2 \left(-\frac{1}{3} - \frac{1}{5} + \frac{1}{2}\right)$$

$$B(0)_z = -4\mu_g\,\sigma.\omega R^2 \left(\frac{-1}{30}\right)$$

$$B(0)_z = +\frac{2\mu_g\,\sigma.\omega R^2}{15} = \frac{2\mu_g\,\sigma 2\pi f R^2}{15} = \frac{\mu_g}{5TR} \cdot \frac{\sigma\, 4\pi R^3}{3}$$

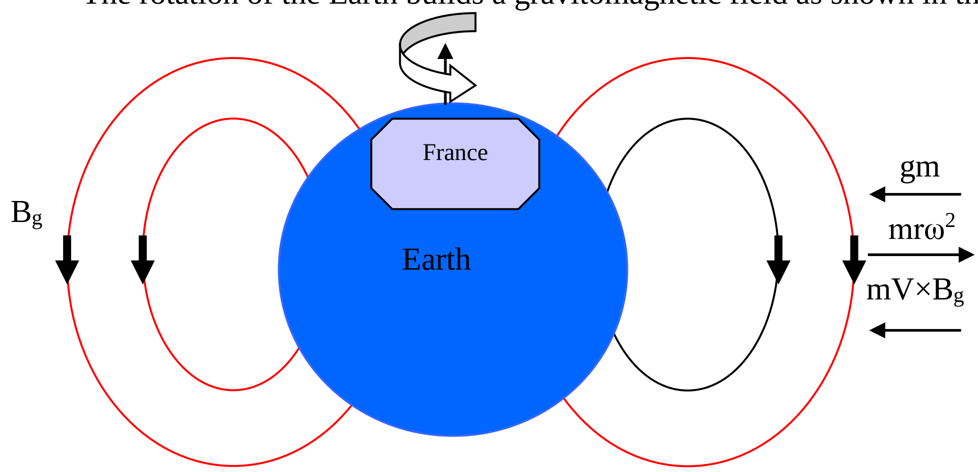
But $\frac{\sigma\, 4\pi R^3}{3}$ = volume density×sphere volume = M masse of the sphere.

$$B(0)_z = \frac{\mu_g M}{5TR}$$

T being the period of rotation.

Gravitomagnetic field in Earths poles

The rotation of the Earth builds a gravitomagnetic field as shown in the following diagram:



The gravitomagnetic field is given by:

$$B(0)_z = \frac{\mu_g M}{5TR} = \frac{2\pi GM}{5Rc^2T}$$, since $\mu_g\varepsilon_g = 1/c^2$ and $G = 1/4\pi\varepsilon_g$

This field $B(0)_z$ is vectored towards the geographic north.
G is the constant of gravity.
T is the period of rotation = 60×60×24 seconds.
$G = 1/4\pi\varepsilon_g$

$\mu_g\varepsilon_g c^2 = 1$

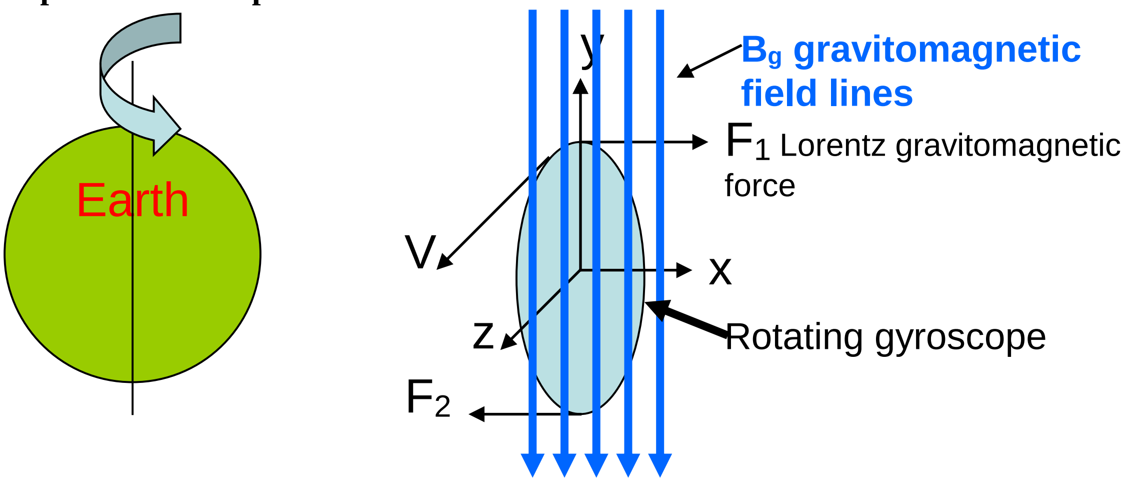
c is the speed of light (hypothesis: The speed of gravitational interaction is equal to the speed of light)
ε_g is the vacuum permittivity of gravitational field, μ_g is the vacuum permeability of gravitomagnetic field in vacuum.

$\mu_g = 9.4 \times 10^{-27}$ m/kg
M = 5.973×10^{24} kg
R = 6.371×10^{6} m
T = 86400 s

$B(0)_z = 2 \times 10^{-14}$ s^{-1}
(units in 1/seconds, just like the frequency)

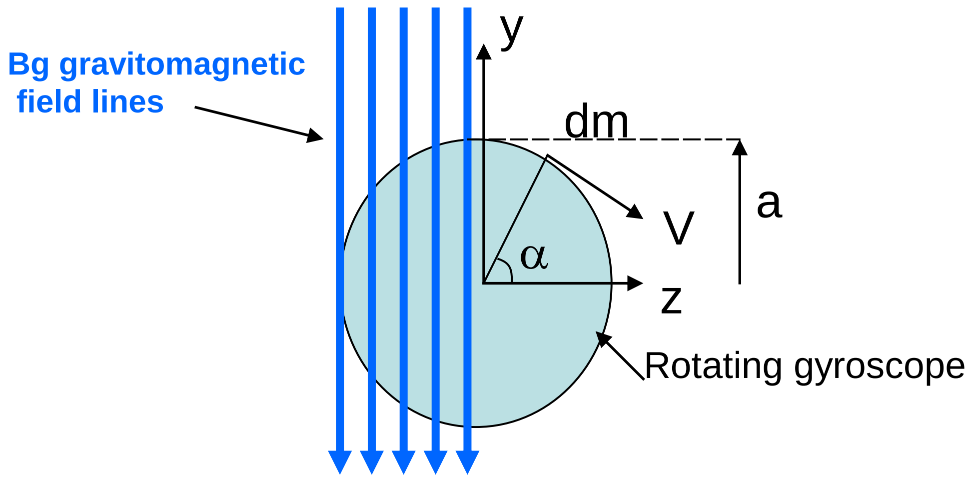
Gravity probe C

How to measure the gravitomagnetic field around the equatorial orbit of the Earth
Experimental setup:



1) The probe should orbit the Earth in equatorial plane as shown in the diagram above and against the direction of the rotation of the Earth. Due to orbiting effect, correction must be done on the obtained gravitomagnetic field.
2) The initial axis of the gyroscope rotation should be perpendicular to the rotation of the Earth to maximize the initial gravitomagnetic couple as the masse cuts the gravitomagnetic field lines.
3) The probe should kept pointing at polar fixed stars.

Lets us calculate the gravitomagnetic couple as shown in the diagram below



The elementary Lorentz gravitomagnetic force dF exerted on an elementary masse dm moving at a velocity of V by the gravitomagnetic field of the Earth Bg is given by;

dF = dmV× Bg, since the gyroscope is very small compared to the Earth we can assume that the gravitomagnetic field of the Earth is horizontal and uniform at the equatorial plane neighbourhood. For simplicity purposes we can assume that the masse of the spinning part of the gyroscope is concentrated in a ring masse. When the gyroscope spins the ring masse cuts the gravitomagnetic field lines an it experiences a Lorentz gravitomagnetic force.

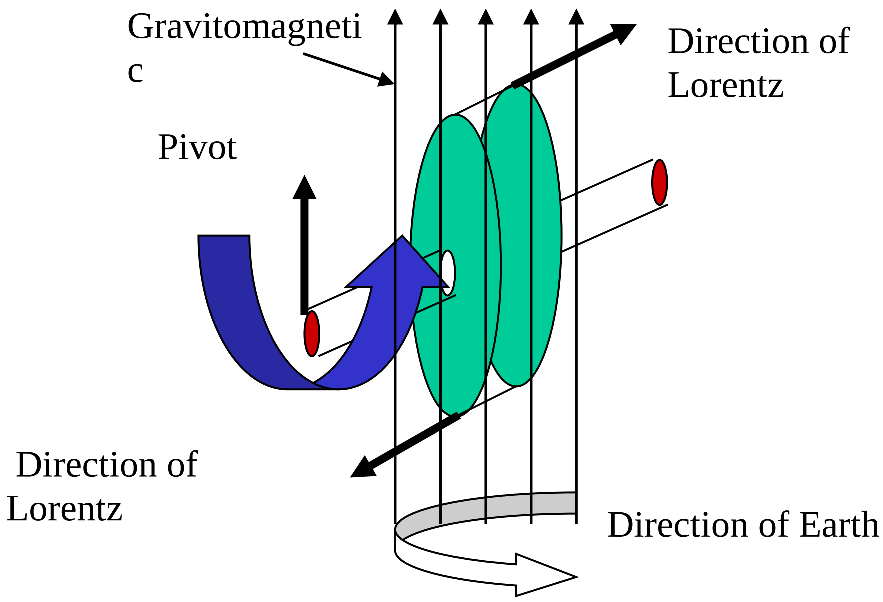
$dF = dm\, V \times B_g$
$dm = \rho a d\alpha$ where ρ is the masse linear density
With the total masse of the gyroscope $m = 2\pi a\rho$

$$V = V_0 \begin{pmatrix} 0 \\ -\cos(\alpha) \\ \sin(\alpha) \end{pmatrix} \qquad B_g = B_0 \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$V \times B_g = V_0 \begin{pmatrix} 0 \\ -\cos(\alpha) \\ \sin(\alpha) \end{pmatrix} \times B_0 \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = V_0\, B_0 \begin{pmatrix} \mathrm{Sin}(x) \\ 0 \\ 0 \end{pmatrix}$$

Gravitomagnetic couple = $ma^{2\omega}B_0$

Measuring the gravitomagnetic field in the North Pole

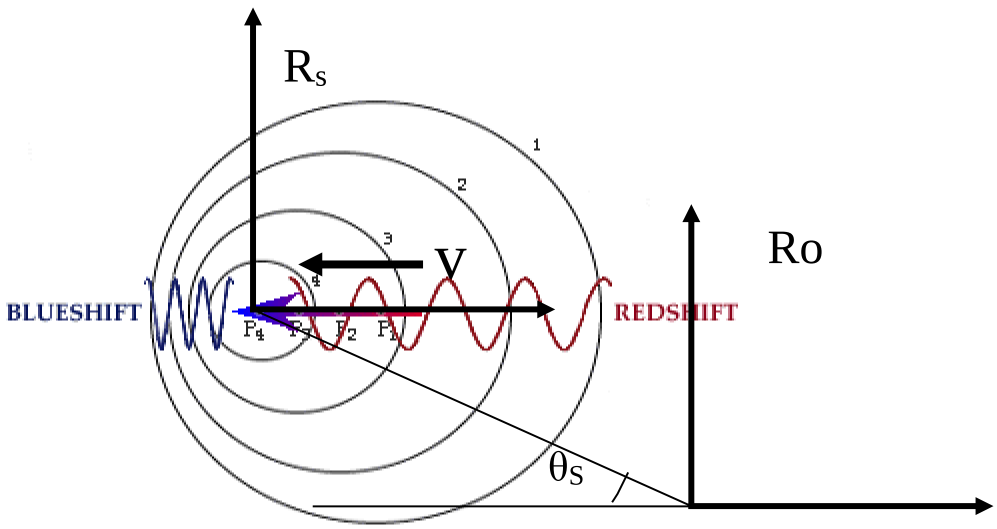


The diagram above shows part of a gyroscope spinning on the North Pole of the Earth, its axis of is horizontal in order to maximize Lorentz couple. A system on which the gyroscope is fixed has a vertical axis of rotation that passes at the middle of the gyroscope. This system rotates in the opposite direction of the Earths rotation at frequency of one cycle par day on order to compensate the rotation of the Earth. When matter cuts the gravitomagnetic field lines, the upper part of the spinning matter is pulled to the right and the lower part of the spinning matter is pulled to the left. Due to the Lorentz couple the gyroscope will pivot in the clockwise direction as shown in the diagram above in the same way as electric motor does in magnetic field in a similar configuration. The trouble is that the gravitomagnetic field is very weak and it would take a long time to have a reasonable deviation on the gyroscope. A better way to do the measurement is to make a differential measurement by simulating a fast rotating Earth. First, the system on which the gyroscope is fixed is spun for a number of times in on direction and then spun for the same number of times in the opposite; the rotation axis of the system on which the gyroscope is fixed should always be in the vertical direction. The resulting gyroscope deviation is the used to measure the gravitomagnetic field on the Earths North Pole. ***Care must be taken not to rotate the system too fast because the gyroscope might start vibrating causing mechanical damage to the system.*** That could be the mission of the Gravity Probe C.

Black holes thermodynamics

Doppler Effect on the electric field

We shall first apply our approach on the electric field and use the experimental results to show its validity, and since the electric field and the gravitational field are governed by the same geometrical and propagation laws, then we will safely apply our approach on the force exerted by the gravitational field on matter to show how matter can escape from the black hole and how matter radiates gravitational waves including the recently discovered B-flat "sound waves". Our results will be in contradiction with the surface at the ***Schwarzschild radius*** which is said to act as **an event horizon** in a static body (point of no return). The trouble with the Schwarzschild radius is that it was calculated using a staic gravitational field without taking into account the Doppler Effect (red shift / blue shift) on the gravitational field. One can use Newton's equation of gravity to get the Schwarzschild radius, this was done by John Mitchell, an English geologist in 1783. But we know that Newton's laws do not hold at relativistic speeds. We shall show, by using Doppler Effect, that the ***horizon of event does not exist*** as long as matter could approach the speed of light, we shall also show that the so called B-flat sound waves are in reality gravitational waves.



By using the Lorentz transformation to determine the Doppler Effect, taking f_S to be the frequency of the source, f_0 the observed frequency and θ_S the sight angle, the relationship between fs and fo is given by;

$$f_0 = f_S.\gamma.(1 - \beta\cos(\theta_S))$$

Where; $\beta = \frac{\upsilon}{c}$ $\quad \gamma = \frac{1}{\sqrt{(1-\beta^2)}}$